0.1 MATHS 201 C.A 2016/2017

1. $y = \tan^{-1}x$ then $(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{du}{dx}$

$y = \tan^{-1}x \quad \frac{dy}{dx} = \frac{1}{1+x^2}$

Using quotient rule

$\frac{d^2y}{dx^2} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

$= \frac{(1+x^2)(0) - 1(2x)}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2}$

$(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{du}{dx}$

$= (1+x^2)\frac{2x}{(1+x^2)^2} + 2x.\frac{1}{1+x^2}$

$= \frac{-2x}{1+x^2} + \frac{2x}{1+x^2} = 0 \quad$ (C)

2. $y = 2xe^{-3x}$ then $\frac{d^2y}{dx^2} + 6\frac{dy}{dx}$

$\frac{dy}{dx} = 2x(-3e^{-3x}) + e^{-3x}(2)$

$= -6xe^{-3x} + 2e^{-3x}$

$\frac{d^2y}{dx^2} = -6x(-3e^{-3x}) + e^{-3x}(-6) - 6e^{-3x}$

$= 18xe^{-3x} - 12e^{-3x}$

$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 18xe^{-3x} - 12e^{-3x} + 6(-6xe^{-3x} + 2e^{-3x})$

$= 18xe^{-3x} - 12e^{-3x} - 36xe^{-3x} + 12e^{-3x}$

$-18xe^{-3x} = 9(-2xe^{-3x})$

but $\quad y = 2xe^{-3x}$

$\therefore \quad = -9y \qquad$ (A)

3. $\int_0^4 \frac{dx}{\sqrt{16-x^2}}$

from standard integral

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(\frac{x}{a}) + c$

comparing

$\int_0^4 \frac{dx}{\sqrt{16-x^2}} = \left[\sin^{-1}\frac{x}{4}\right]_0^4$

$= \left[\sin^{-1}\frac{4}{4} + c\right] - \left[\sin^{-1}\frac{0}{4} + c\right]$

$\sin^{-1}1 - \sin^{-1}0 = \frac{\pi}{2} \qquad$ (C)

4. $y = \tan^{-1}(\frac{\sin t}{\cos t - 1})$ then $\frac{du}{dx}$ is

Let $U = \frac{\sin t}{\cos t - 1}$

$y = \tan^{-1}U \quad \frac{dy}{du} = \frac{1}{1+U^2}$

from $\quad U = \frac{\sin t}{\cos t - 1} \quad$ using quotient rule

$\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

$\therefore \frac{du}{dt} = \frac{(\cos t - 1)(\cos t) - (\sin t)(-\sin t)}{(\cos t - 1)^2}$

$\frac{\cos^2 t - \cos t + \sin^2 t}{(\cos t - 1)^2}$

$\frac{\cos^2 t + \sin^2 t - \cos t}{(\cos t - 1)^2} \quad$ but $\cos^2 t + \sin^2 t = 1$

$= \frac{1 - \cos t}{(\cos t - 1)^2}$

$\therefore \frac{dy}{dx} = \frac{1}{1 + (\frac{\sin t}{\cos t - 1})^2} \times \frac{1 - \cos t}{(\cos t - 1)^2}$

$\frac{dy}{dx} = \frac{1}{1 + \frac{\sin^2 t}{(\cos t - 1)^2}} \times \frac{1 - \cos t}{(\cos t - 1)^2}$

$\frac{1}{\frac{(\cos t - 1)^2 + \sin^2 t}{(\cos t - 1)^2}} \times \frac{1 - \cos t}{(\cos t - 1)^2}$

$= \frac{(\cos t - 1)^2}{(\cos t - 1)^2 + \sin^2 t} \times \frac{1 - \cos t}{(\cos t - 1)^2}$

$= \frac{1 - \cos t}{\cos^2 t - 2\cos t + 1 + \sin^2 t} = \frac{1 - \cos t}{\cos^2 t + \sin^2 t - 2\cos t}$

$\frac{1 - \cos t}{1 + 1 - 2\cos t} = \frac{1 - \cos t}{2 - 2\cos t}$

$= \frac{1 - \cos t}{2(1 - \cos t)} = \frac{1}{2} \qquad$ (B)

5. $x^2 + y^2 - 2x - 2y = 3$ at $x = 2$

substituting the value of $x = 2$

$2^2 + y^2 - 2(2) - 2y = 3$

$4 + y^2 - 4 - 2y = 3$

$y^2 - 2y - 3 = 0$

factoring

$(y+1)(y-3)$
$y=-1$ or $y=3$
point of y should be positive in these case
$\therefore\ x=2\quad y=3$
solving the gradient
$x^2+y^2-2x-2y=3$
differentiating implicitly

$$2x+2y\frac{dy}{dx}-2-2\frac{dy}{dx}=0$$

$$(2y-2)\frac{dy}{dx}=-2x+2$$

$$\frac{dy}{dx}=\frac{-2x+2}{2y-2}\Big|_{x=2\ y=3}$$

$$=\frac{-2(2)+2}{2(3)-2}=\frac{-4+2}{6-2}=\frac{-2}{4}=\frac{-1}{2}(M)$$

$$Tangent=\frac{-1}{gradient\ of\ normal}$$

$$T=\frac{-1}{N}\implies T=M$$

$\therefore$ Gradient of the tangent $=\frac{-1}{2}$

6. $\int \sin^2 x\cos^5 x\,dx=\int \sin^2 x(\cos^4 x)\cos x\,dx$
$\int \sin^2 x(1-\sin^2 x)^2\cos x\,dx$
$\int \sin^2 x\cos x(1-2\sin^2 x+\sin^4 x)dx$
$=\int(\sin^2 x\cos x-2\sin^4 x\cos x+\sin^6 x\cos x)dx$
integrating

$$=\frac{\sin^3 x}{3}-\frac{2}{5}\sin^5 x+\frac{\sin^7 x}{7}+C$$

$$\frac{1}{7}\sin^7 x-\frac{2}{5}\sin^5 x+\frac{1}{3}\sin^3 x+C\qquad (B)$$

7. $\int e^{2x}\cos 3x\,dx$
from standard integral

$$\int e^{ax}\cos bx=\frac{e^{ax}}{a^2+b^2}(b\sin bx+a\cos bx)$$

$a=2\quad b=3$
comparing

$$\int e^{2x}\cos 3x\,dx=\frac{e^{2x}}{2^2+3^2}(3\sin 3x+2\cos 3x)$$

$$=\frac{e^{2x}}{13}(3\sin 3x+2\cos 3x)\qquad (E)$$

8. $\frac{dy}{dx}$ of $y^2-\cos 2x=?$
differentiating implicitly

$$2y\frac{dy}{dx}-(-2\sin 2x)=0$$

$$2y\frac{dy}{dx}+2\sin 2x=0$$

$$\frac{dy}{dx}=-\frac{2\sin 2x}{2y}\Big|_{\frac{\pi}{4},\,-1}$$

$$\frac{dy}{dx}=\frac{-2\sin 2(-1)}{2(\frac{\pi}{4})}$$

$$=\frac{2\sin 2}{90}=\frac{0.0698}{90}=0.00077\qquad (E)$$

9. $y=\tanh^{-1}\left(\frac{1-x}{1+x}\right)$ then $2x\frac{dy}{dx}$ is
Let $U=\frac{1-x}{1+x}$
$y=\tanh^{-1}(U)$ using quotient rule

$$\frac{dy}{du}=\frac{V\frac{du}{dx}-U\frac{dv}{dx}}{V^2}$$

$$\frac{(1+x)(-1)-(1-x)(1)}{(1+x)2}$$

$$=\frac{-1-x-(1-x)}{(1+x)^2}$$

$$\frac{dy}{dx}=\frac{-1-x-1+x}{(1+x)^2}=\frac{-2}{(1+x)^2}$$

$$=\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

$$=\frac{1}{1-U^2}\times\frac{-2}{(1+x)^2}$$

$$\frac{dy}{dx}=\frac{(1+x)^2}{(1+x)^2-(1-x)^2}\times\frac{-2}{(1+x)^2}$$

$$\frac{-2}{(1+x)^2-(1-x)^2}=\frac{-2}{4x}=\frac{-1}{2x}$$

$$\frac{dy}{dx}=\frac{-1}{2x}$$

from the condition

$$2x\frac{dy}{dx}=2x\times\frac{-1}{2x}=-1\qquad (B)$$

10. If $y=\tan^{-1}(\frac{x}{2})$

Scanned by CamScanner

$y = \tan^{-1} U \quad U = \frac{x}{2}$

$\frac{du}{dx} = \frac{2(1) - x(0)}{2^2} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{dy}{du} = \frac{1}{1+U^2}$

$= \frac{1}{1+(\frac{x}{2})^2} = \frac{1}{\frac{1}{1}+\frac{x^2}{4}} = \frac{1}{\frac{4+x^2}{4}} = \frac{4}{4+x^2}$

$\frac{dy}{dx} = \frac{4}{4+x^2} \times \frac{1}{2} = \frac{2}{4+x^2}$

then $(4+x^2)\frac{dy}{dx}$

$= 4 + x^2 \times \frac{2}{4+x^2} = 2 \qquad$ (D)

11. $y^2 + x^2 = 144 \quad (4, 12)$

differentiating implicitly

$2y\frac{dy}{dx} + 2x = 0$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}|_{4,12} = \frac{-4}{12} = \frac{-1}{3}(M)$

Normal $= \frac{-1}{gradient\ of\ tangent}$

$= \frac{-1}{\frac{-1}{3}} = 3$

equation of normal

$= y - y_1 = m(x - x_1)$

$= y - 12 = 3(x-4)$

$y - 12 = 3x - 12$

$y = 3x \qquad$ (E)

12. $y = ln \sec x$

Let $U = \sec x \qquad \frac{du}{dx} = \sec x \tan x$

$y = lnU \qquad \frac{dy}{du} = \frac{1}{U}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{U} \times \sec x \tan x$

$= \frac{1}{\sec x} \times \sec x \tan x$

$\frac{dy}{dx} = \tan x$

from the condition

$cosecx\frac{dy}{dx} = ?$

$\frac{dy}{dx} = \tan x = \frac{\sin x}{\cos x} \quad$ divide both side by $\sin x$

$\frac{1}{\sin x}\frac{dy}{dx} = \frac{1}{\cos x}$

Remember $\frac{1}{\sin x} = cosecx$ and $\frac{1}{\cos x} = \sec x$

$\therefore \quad cosecx\frac{dy}{dx} = \sec x \qquad$ (B)

13. $\int_0^{\frac{\pi}{2}} 2\sin^2 x dx$

from trigonometry

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$2\int \frac{1}{2}(1-\cos 2x)dx$

$2 \times \frac{1}{2}\int 1 - \cos 2x dx$

$\int_0^{\frac{\pi}{2}} 1 - \cos 2x = \left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$

$= \left[\frac{\pi}{2} - \frac{1}{2}\sin 2\frac{\pi}{2}\right] - \left[0 - \frac{1}{2}\sin 2(0)\right]$

$= \frac{\pi}{2} - \frac{1}{2}\sin \pi = \frac{\pi}{2} \qquad$ (E)

14. $e^{\frac{x}{2}} = a + bx + cx^2 + dx^3$ the value of $d = ?$

Maclaurins series

$= f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4}f^{iv}(0) + - - - - - -$

$f(x) = e^{\frac{x}{2}}$

$f'(x) = \frac{e^{\frac{x}{2}}}{2} = f'(0) = \frac{1}{2}$

$f''(x) = \frac{e^{\frac{x}{2}}}{4} = f''(0) = \frac{1}{4}$

$f'''(x) = \frac{e^{\frac{x}{2}}}{8} = f'''(0) = \frac{1}{8}$

$f^{iv}(x) = \frac{e^{\frac{x}{2}}}{16} = f^{iv}(0) = \frac{1}{16}$

By comparison

$dx^3 = \frac{x^3}{3!} \times \frac{1}{8}$

$d = \frac{1}{3 \times 2 \times 8} = \frac{1}{48} \qquad$ (C)

15. $\int_0^{\frac{\pi}{2}} x\cos x dx$

Using integral by part

$u = x \quad du = dx$

$dv = \cos x \quad v = \sin x$

$= uv - \int vdu$

$= x\sin - \int \sin x$

$= x\sin x - (-\cos x) = x\sin x + \cos x|_0^{\frac{\pi}{2}}$

$\left[\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right] - [0\sin 0 + \cos 0]$

$\frac{\pi}{2} \times 1 + 0 - 0 - 1$

$= \frac{\pi}{2} + 0 - 0 - 1 = \frac{\pi}{2} - 1$ (B)

0.2 MATHS 201 2015/2016 EXAMINATION

In questions 1 - 14, obtain the first derivative of the function indicated

1. $y = \frac{\cos t}{\cos t + \sin t}$ (A) $\frac{-2\cos t\sin t}{1+\sin 2t}$ (B) $\frac{1+\cos t}{1+\sin 2t}$ (C) $\frac{1}{1+\sin 2t}$ (D) $\frac{2+\cos t\sin t}{1+\sin t}$

Solution

$y = \frac{\cos t}{\cos t + \sin t}$ using quotient rule

$u = \cos t \quad \frac{du}{dt} = -\sin t$

$v = \cos t + \sin t \quad \frac{dv}{dt} = \cos t - \sin t$

$\frac{dy}{dt} = \frac{V\frac{du}{dt} - U\frac{dv}{dt}}{V^2}$

$\frac{dy}{dt} = \frac{(\cos t + \sin t)(-\sin t) - \cos t(\cos t - \sin t)}{(\cos t + \sin t)^2}$

$= \frac{\sin t\cos t - \sin^2 t - \cos^2 t + \sin t\cos t}{\cos^2 t + \sin^2 t + 2\sin t\cos t}$

$= \frac{-(\sin^2 t + \cos^2 t)}{(\cos^2 t + \sin^2 t) + 2\sin t\cos t}$

Recall $\sin^2 t + \cos^2 t = 1$ and $\sin 2t = 2\sin t\cos t$

$\therefore = \frac{-1}{1 + \sin 2t}$

Hence $\frac{dy}{dx} = \frac{-1}{1 + \sin 2t}$ no answer

2. $y = \frac{2t^3}{2 + 3t^3}$ (A) $\frac{12t^3}{(2+3t^3)^2}$ (B) $\frac{1+t^2}{(}2 + 3t^3)^2$ (C) $\frac{3t}{(2+3t^3)^2}$ (D) $-\frac{1+3t^2}{2+3t^3}$

Solution

Let $u = 2t^3 \quad \frac{du}{dt} = 6t^2$

$v = 2 + 3t^3 \quad \frac{dv}{dt} = 9t^2$

$\frac{dy}{dt} = \frac{V du - U dv}{V^2}$

$= \frac{(2 + t^3)(6t^2) - 2t^3(9t^2)}{(2 + 3t^3)^2}$

$\frac{12t^2 + 18t^3 - 18t^3}{(2 + 3t^3)^2}$

Scanned by CamScanner

$$\frac{dy}{dt} = \frac{12t^2}{(2+3t^3)^2} \quad (A)$$

3. $y = \sin^2 t \cos 2t$ (A) $2\sin 2t \cos 2t$ (B) $\sin 4t - \sin 2t$ (C) $6 \sin 2t \cos 2t$ (D)$4\sin 2t - \cos 2t$

Solution

Let $u = \sin^2 t \quad \frac{du}{dt} = 2\sin t \cos t$

$v = \cos 2t \quad \frac{dv}{dt} = -\sin 2t$

$\frac{dy}{dt} = Udv + Vdu$

$= (\sin^2 t)(-2\sin 2t) + (\cos 2t)(2 \sin t \cos t)$

$= -2\sin^2 t \sin t + 2 \sin t \cos t \cos 2t$

Recall, $\sin 2t = 2 \sin t \cos t$

$\frac{dy}{dt} = -2\sin^2 t \sin 2t + \sin 2t \cos 2t$

Recall $\cos 2t = 1 - 2\sin^2 t$

$\frac{dy}{dx} = (\cos 2t - 1)\sin 2t + \sin 2t + \cos 2t$

$= \sin 2t \cos 2t - \sin 2t + \sin 2t \cos 2t$

$= 2 \sin 2t \cos 2t - \sin 2t$

Recall, $\sin 4t = 2\sin 2t \cos 2t$

$\therefore \quad \frac{dy}{dt} = \sin 4t - \sin 2t \quad$ (B)

4. $y = \cos^2(\frac{a}{t})$ (a is constant) (A) $2\sin^2(\frac{a}{t})\cos(\frac{a}{t})$ (B) $-\frac{a}{t^2}\sin(\frac{2a}{t})$ (C) $\frac{1}{t^2}\sin(\frac{2a}{t})$ (D) $\frac{a}{t^2}\cos(\frac{2a}{t})$

Solution

$c = \cos u \quad u = \frac{a}{t} = at^{-1}$

$\frac{du}{dt} = -at^{-2} = -\frac{a}{t^2}$

$\therefore \quad y = \cos^2 u$

$v = \cos u \quad \frac{dv}{du} = -\sin u$

$y = v^2 \quad \frac{dy}{dv} = 2v$

$\therefore \quad \frac{dy}{dt} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dt}$

$= 2v \times (-\sin u) \times (at^{-2})$

$\frac{dy}{dt} = \frac{2av \sin u}{t^2}$

Replacing the value of v and u

$\frac{dy}{dt} = \frac{2a \cos u \sin u}{t^2} = \frac{a \sin 2u}{t^2}$

$\therefore \quad \frac{dy}{dt} = \frac{-a}{t^2}\sin(\frac{2a}{t}) \quad$ (B)

5. $y = 2\tan x + \tan^2 x$ (A)$\sec^2 x + \tan x$ (B)$\tan^4 x + 1$ (C)$\tan^2 x + \sec^2 x$ (D)$\sec^2 x(1 + \tan x)$

Solution

Let $u = 2\tan x \quad \frac{du}{dx} = 2\sec^2 x$

$v = \tan^2 x \quad \frac{dv}{dx} = 2 \tan x \sec^2 x$

$\frac{dy}{dt} = \frac{du}{dx} \times \frac{dv}{dx}$

$= 2\sec^2 x + 2\tan x \sec^2 x$

$= 2\sec^2 x(1 + \tan x)$ No Ans

6. $y = e^{2t}\tan^{-1} t$ (A)$[e^{2t}(\tan 2t - 1)]^2$ (B) $e^{2t}\tan 2t(1+4t^2)^{-1}$ (C) $[e^{2t}(\sec^{-1} 2t)]^2$ (D)$2e^{2t}[\tan^{-1} 2t + (1+4t^2)^{-1}]$

Solution

Let $u = e^{2t} \quad \frac{du}{dt} = 2e^{2t}$

$v = \tan^{-1} t \quad \frac{dv}{dt} = \frac{1}{1+t^2}$

$\frac{dy}{dt} = Udv + Vdu$

$= e^{2t}(\frac{1}{1+t^2}) + (\tan^{-1} t)(2e^{2t})$

$= \frac{e^{2t}}{1+t^2} + 2e^{2t}\tan^{-1} t$

$\frac{dy}{dt} = e^{2t}(\frac{1}{1+t^2} + 2\tan^{-1} t)$ No Ans

7. $y = ln\sqrt{\frac{2x-1}{2x+1}}$ (A)$\frac{2x}{2x+1}$ (B)$\frac{x}{(2x-1)^2}$ (C)$\frac{2}{4x^2-1}$ (D)$\frac{2}{(2x+1)^2}$

Solution

$y = lnv \quad v = \sqrt{u} \quad u = \frac{2x-1}{2x+1}$

$\frac{du}{dx} = \frac{(2x+1)2 - (2x-1)2}{(2x+1)^2}$

$v = \sqrt{u} = u^{\frac{1}{2}}$

$\frac{dv}{du} = \frac{1}{2}U^{\frac{-1}{2}} = \frac{1}{2\sqrt{u}}$

$y = lnv \quad \frac{dy}{dv} = \frac{1}{v}$

$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dx}$

$$= \frac{1}{v} \times \frac{1}{2\sqrt{u}} \times \frac{4x+2-4x+2}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{1}{v} \times \frac{1}{2\sqrt{u}} \times \frac{4}{(2x+1)^2}$$

$$= \frac{1}{u} \times \frac{2}{(2x+1)^2}$$

$$= \frac{1}{\frac{2x-1}{2x+1}} \times \frac{2}{(2x+1)^2} = \frac{2x+1}{2x-1} \times \frac{2}{(2x+1)^2}$$

$$= \frac{2}{(2x)^2 - 1^2} = \frac{2}{4x-1}$$

$$\frac{dy}{dx} = \frac{2}{4x-1} \quad \text{(C)}$$

8. $y = ln \tan^2 x$ (A)$\sec^2 x - 1$ (B)$2(\cot x + \tan x)$ (C)$\sec x + \tan x$ (D)$cosecx + \cot x$

Solution

Let $u = \tan x \quad \frac{du}{dx} = \sec^2 x$

$v = u^2 \quad \frac{du}{dv} = 2u$

$y = lnv \quad \frac{dy}{dv} = \frac{1}{v}$

$$\frac{dy}{dx} = \frac{du}{dv} \times \frac{dv}{du} \times \frac{du}{dx}$$

$$= \frac{1}{v} \times 2u \times \sec^2$$

$$= \frac{1}{u^2} \times 2u \times \sec^2 x = \frac{2\sec^2 x}{u}$$

$$\frac{dy}{dx} = \frac{2\sec^2 x}{\tan x} = \frac{2(\tan^2 x + 1)}{\tan x}$$

$$= 2(\tan x + \cot x) \quad \text{(B)}$$

9. $y = lncosecx$ (A)$-\cot x$ (B)$\tan x$ (C)$-\cos x$ (D)$cosecx$

Solution

Let $u = cosecx \quad \frac{du}{dx} = cosecx \cot x$

$y = lnu \quad \frac{dy}{du} = \frac{1}{u}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u} \times -cosecx \cot x$$

$$\frac{dy}{dx} = \frac{1}{cosecx} \times -cosecx \cot x$$

$$\therefore \frac{dy}{dx} = -\cot x \quad \text{(A)}$$

10. $y = ln\sqrt{\frac{x^2-2}{x^2+2}}$ (A)$\frac{8x}{x^4-4}$ (B)$\frac{4x}{(x^2+1)^2}$ (C)$\frac{2x}{(x^2-1)^2}$ (D)$\frac{8x}{x^2+1}$

Solution

Let $U = \frac{x^2-2}{x^2+2}, \quad \frac{du}{dx} = \frac{(x^2+2)2x - (x^2-2)2x}{(x^2+2)^2}$

$V = \sqrt{U} = U^{\frac{1}{2}} \quad \frac{dv}{du} = \frac{1}{2\sqrt{U}}$

$y = lnv \quad \frac{dy}{dv} = \frac{1}{V}$

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dx}$$

$$\frac{1}{V} \times \frac{1}{2\sqrt{U}} \times \frac{8x}{(x^2+2)^2}$$

$$\frac{1}{U} \times \frac{4x}{(x^2+2)^2}$$

$$= \frac{1}{\frac{x^2-2}{x^2+2}} \times \frac{4x}{(x^2+2)^2} = \frac{x^2+2}{x^2-2} \times \frac{4x}{(x^2+2)^2}$$

$$\frac{dy}{dx} = \frac{4x}{x^4-4} \quad \text{No Ans}$$

11. $y = \sin x + x\cos y$ (A)$\frac{\sin y}{1-\cos y}$ (B)$\frac{\cos x+\cos y}{1+x\sin y}$ (C)$\frac{\cos y+\sin x}{1-x\sin y}$ (D)$\frac{\cos y}{1+x\cos y}$

Solution

$$\frac{dy}{dx} = \cos x + x\sin y\frac{dy}{dx} + \cos y$$

$$\frac{dy}{dx} - x\sin y\frac{dy}{dx} = \cos x + \cos y$$

$$\frac{dy}{dx}(1 - x\sin y) = \cos x + \cos y$$

$$\frac{dy}{dx} = \frac{\cos x + \cos y}{1 - x\sin y} \quad \text{No Ans}$$

12. $y = \tanh^{-1}(\frac{1-2x}{1+2x})$ (A)$\frac{-2}{1+2x}$ (B)$\frac{-2x}{1+2x}$ (C)$\frac{-1}{2x}$ (D)$\frac{1}{2(x+2)}$

Solution

Let $U = \frac{1-2x}{1+2x} \quad \frac{du}{dx} = \frac{(1+2x)(-2) - (1-2x)(2)}{(1+2x)^2}$

$y = \tanh^{-1} U \quad \frac{dy}{du} = \frac{1}{1-U^2}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{1-U^2} \times \frac{-2-4x-2+4x}{(1+2x)^2}$$

$$\frac{dy}{dx} = \frac{1}{1-(\frac{1-2x}{1+2x})^2} \times \frac{-4}{(1+2x)^2}$$

$$= \frac{1}{\frac{(1+2x)^2-(1-2x)^2}{(1+2x)^2}} \times \frac{-4}{(1+2x)^2}$$

$$= \frac{(1+2x)^2}{1+4x+4x^2-(1-4x+4x^2)} \times \frac{-4}{(1+2x)^2}$$

$$\frac{dy}{dx} = \frac{-4}{1+4x+4x^2-1+4x-4x^2}$$

$$= \frac{-4}{8x} = \frac{-1}{2x} \quad (C)$$

13. $y = sech^{-1}(\sin x)$ (A)$sechx$ (B)$coshecx$ (C)$-\sec x$ (D)$cosecx$

Solution

Let $y = sech^{-1}U \quad \frac{dy}{du} = \frac{-1}{U\sqrt{1-U^2}}$

$U = \sin x \quad \frac{dy}{dx} = \cos x$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{-1}{U\sqrt{1-U^2}} \times \cos x$$

$$= \frac{-\cos x}{\sin x\sqrt{1-\sin^2 x}}$$

$$= \frac{-\cos x}{\sin x\sqrt{\cos^2 x}} = \frac{-\cos x}{\sin x \cos x} = \frac{-1}{\sin x} = -cosecx$$

$\therefore \frac{dy}{dx} = -cosecx$ No Ans

14. $y = cosech^{-1}(\tan x)$ (A)$\cot x$ (B)$\tan x$ (C)$\tanh x$ (D)$\coth x$

Solution

Let $u = \tan x \quad \frac{du}{dx} = \sec^2 x$

$y = cosech^{-1}U \quad \frac{dy}{du} = \frac{-1}{U\sqrt{1-U^2}}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{-1}{U\sqrt{1-U^2}} \times \sec^2 x$$

$$\frac{dy}{dx} = \frac{-1}{\tan x\sqrt{1-\tan^2 x}}$$

$$= \frac{-\sec^2 x}{\tan x\sqrt{1-\tan^2 x}}$$

$$= \frac{-\sec^2 x}{\tan x\sqrt{\frac{\cos^2 x-\sin^2 x}{\cos x}}} = \frac{-\sec^2 x . \cos x}{\tan x\sqrt{\cos 2x}}$$

$$\frac{-\sec x}{\tan x\sqrt{\cos 2x}} = \frac{-1}{\cos x} \times \frac{\cos x}{\sin x\sqrt{\cos 2x}}$$

$$\frac{-1}{\sin x\sqrt{\cos 2x}} = \frac{-cosecx}{\sqrt{\cos 2x}}$$ No Answer

Perform the indicated operation in questions 15 -21

15. $\int \frac{1}{\sqrt{2x+1}} \sin\sqrt{2x+1}dx$ (A)$2\sin\sqrt{2x+1}+c$ (B)$-2\sin\sqrt{2x+1}+c$ (C)$2\cos\sqrt{2x+1}+c$ (D)$-\cos\sqrt{2x+1}+c$

Solution

Let $u = \sqrt{2x+1} \quad \frac{du}{dx} = 2 \times \frac{1}{2}(2x+1)^{\frac{-1}{2}}$

$dx = \sqrt{2x+1} \quad du = udu$

$\implies \int \frac{1}{u}\sin u.udu = \int \sin udu$

$-\cos u + c = -\cos(\sqrt{2x+1}) + c$ (D)

16. $\int_0^{\frac{\pi}{2}} e^{2x}\cos 3xdx$ (A)$\frac{e^{\pi}}{4}$ (B)$\frac{3e^{\pi}}{25}$ (C)$\frac{-3e^{\pi}}{13}$ (D)$\frac{2e^{\pi}}{13}$

Solution

Let $u = \cos 3x \quad du = -3\sin 3x$

$dv = e^{2x} = v = \frac{1}{2}e^{2x}$

$\int udv = uv - \int vdu$

$\int e^{2x}\cos 3xdx = \frac{1}{2}e^{2x}\cos 3x - \int \frac{1}{2e^{2x}} - 3\sin 3x$

$= \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}\int e^{2x}\sin 3x \quad ------ (i)$

Now $\int e^{2x}\sin 3x$

Let $u = \sin 3x \quad du = 3\cos 3x$

$dv = e^{2x} \quad v = \frac{1}{2}e^{2x}$

$\int e^{2x}\sin 3x = \frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\int e^{2x}\cos 3x \quad ------ (ii)$

Now substitute equation (ii) into (i)

$\int_0^{\frac{\pi}{2}} e^{2x}\cos 3xdx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}[\frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\int e^{2x}\cos 3xdx]$

$\int_0^{\frac{\pi}{2}} e^{2x}\cos 3xdx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x - \frac{9}{4}\int e^{2x}\cos 3xdx$

$[1+\frac{9}{4}]\int_0^{\frac{\pi}{2}} e^{2x}\cos 3xdx = \frac{e^{2x}}{2}(\cos 3x + \frac{3}{2}\sin 3x)$

$\int_0^{\frac{\pi}{2}} e^{2x}\cos 3xdx = [\frac{4}{26}e^{2x}(\cos 3x + \frac{3}{2}\sin 3x)]_0^{\frac{\pi}{2}}$

$$\left[\frac{4e^{2\times\frac{\pi}{2}}}{26}\left(\cos\frac{3\pi}{2} + \frac{3}{2}\sin\frac{3\pi}{2}\right)\right] - \left[\frac{4e^0}{26}(\cos 0 + \frac{3}{2}\sin 0)\right]$$

$$\frac{4e^{\pi}}{26}\left[0+\frac{3}{2}(-1)\right] - \left[\frac{4}{26}(1+0)\right]$$

$$\frac{4e^{\pi}}{26}\left(\frac{-3}{2}\right) - \frac{4}{26} = \frac{10e^{\pi}}{26} - \frac{4}{26}$$

$$\frac{-2(3e^{\pi}+2)}{26} = \frac{-(3e^{\pi}+2)}{13}$$

$$\frac{-1}{13}(3e^{\pi}+2) = \frac{-3e^{\pi}}{13} - \frac{2}{13}$$ No answer

17. $\int_0^{\infty} x^2e^{-ax}dx, \ a>0$ (A)$3a^{-2}$ (B)$2a^{-3}$ (C)$-2a^{-2}$ (D)$-2a^{-3}$

Solution

Let $u = x^2 \quad \frac{du}{dx} = 2x$

$dv = e^{-ax} \quad v = \frac{-1}{a}e^{-ax}$

$\int_0^\infty x^2 e^{-ax}dx = \frac{-x^2}{a}e^{-ax} - \int \frac{-2x}{a}e^{-ax}$

$= \frac{-x^2}{a}e^{-ax} + \frac{2}{a}\int xe^{-ax}dx$

Now $\int xe^{-ax}dx$

$u = x \quad du = 1$

$dv = e^{-ax} \quad v = \frac{-1}{a}e^{-ax}$

$\int xe^{-ax}dx = \frac{-x}{a}e^{-ax} - \int \frac{-1}{a}e^{-ax} = \frac{-x}{a}e^{-ax} + \frac{1}{a}\int e^{-ax}$

$= \frac{-x}{a}e^{-ax} - \frac{q}{a^2}e^{-ax}$

then $\int_0^\infty x^2 e^{-ax}dx = \frac{-x^2}{a}e^{-ax} + \frac{2}{a}[\frac{-x}{a}e^{-ax} - \frac{1}{a^2}e^{-ax}]$

$= [\frac{-x^2}{a}e^{-ax} - \frac{2x}{a^2}e^{-ax} - \frac{2}{a^3}e^{-ax}]_0^\infty$

$= [\frac{-e^{-ax}}{a}(x^2 + \frac{2x}{a} + \frac{2}{a^2})]_0^\infty$

Recall $\quad e^{-a\times\infty} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$[\frac{-e^{-a\infty}}{a}(\infty^2 + \frac{2\infty}{a} + \frac{2}{a^2})] - [\frac{-e^{-a(0)}}{a}(0^2 + \frac{2(0)}{a} + \frac{2}{a^2})]$

$= [-0(\infty^2 + \infty + \frac{2}{a^2})] - [\frac{-1}{a}(0^2 + 0 + \frac{2}{a^2})]$

$= 0 - [\frac{-1}{a}(\frac{2}{a^2})] = -(\frac{-2}{a^3}) = \frac{2}{a^3}$

$= 2a^{-3} \quad$ (B)

18. $\int \frac{2+2x}{(1+x^2)(1-x)}dx \quad (A)ln(\frac{1+x^2}{1-x})^2 + c \ (B)ln(x-1) + \tan^{-1}x + c$
$(C)ln(\frac{1+x^2}{1-x}) + c \ (D)ln(x-1) + \tan^{-1}x + c$

Solution

$\int \frac{2+2x}{(1+x^2)(1-x)}dx = \int(\frac{Ax+B}{1+x^2}) + (\frac{C}{1-x})dx$

Applying the appropriate method of solving partial fraction

We have $\ A = 2 \ B = 0 \ C = 2$

$\int \frac{2+2x}{(1+x^2)(1-x)}dx = \int \frac{2x}{1+x^2}dx + \int \frac{2}{1-x}dx$

Let $u = 1 + x^2 \quad \frac{du}{dx} = 2x \quad dx = \frac{du}{2x}$

For $\ \int \frac{2x}{u} \times \frac{du}{2x} = lnu = ln(1+x^2) + c$

Let $u = 1 - x \quad \frac{du}{dx} = -1 \quad dx = -du$

$\int \frac{2}{u}(-du) = -2lnu = -2ln(1-x) + c$

Hence, $\quad \int \frac{2+2x}{(1+x^2)(1-x)}dx = ln(1+x^2) - 2ln(1-x) + c$

$= ln(1+x^2) - ln(1-x^2) + c$

$= ln\left[\frac{1+x^2}{1-x^2}\right] + c \quad$ No Answer

19. $\int 2e^{-2x}\sin(e^{-2x})dx \quad (A)-\sin e^{-2x} + c \ (B)\cos e^{-2x} + c \ (C) -2\cos e^{-2x} + c$
$(D)2\sin e^{-2x} + c$

Solution

Let $u = e^{-2x} \quad \frac{du}{dx} = -2e^{-2x} \quad dx = \frac{-du}{2e^{-2x}}$

$\int 2e^{-2x}\sin u.\frac{du}{-2e^{-2x}} = -\int \sin u du$

$\Longrightarrow \quad -\sin u du = -[-\cos u] + c$

$= \cos e^{-2x} + c \quad$ (B)

20. $\int \frac{1}{1+e^{2x}}dx \quad (A)x + \tan^{-1}e^x + c \ (B)x - ln(1+e^{2x}) + c \ (C)ln(1+e^{2x}) + c \ (D)\tan^{-1}e^{2x} + c$

Solution

$\int \frac{1+e^{2x}-e^{2x}}{1+e^{2x}}dx = \int(\frac{1+e^{2x}}{1+e^{2x}} - \frac{e^{2x}}{1+e^{2x}})dx$

$= \int 1dx - \int(\frac{e^{2x}}{1+e^{2x}})dx$

Let $u = e^{2x}, \quad \frac{du}{dx} = 2e^{2x}, \quad dx = \frac{du}{2e^{2x}}$

$= \int dx - \int \frac{e^{2x}}{1+u}.\frac{du}{2e^{2x}}$

$= \int dx - \frac{1}{2}\int \frac{du}{1+u}$

$= x - \frac{1}{2}ln(1+u) + c$

$= x - \frac{1}{2}ln(1+e^{2x}) + c$

$\Longrightarrow \quad x - ln(1+e^{2x})^{\frac{1}{2}} + C \qquad$ No answer

21. $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx \quad (A)\tan^{-1}e^x + c \ (B)\sin^{-1}e^x + c \ (C)\cos^{-1}e^x + c$
$(D)\sec^{-1}e^x + c$

Solution

Let $u = e^x, \quad \frac{du}{dx} = e^x, \quad dx = \frac{du}{e^x}$

$\int \frac{e^x}{\sqrt{1-u^2}}.\frac{du}{e^x} = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}u + c$

$\therefore \quad \int \frac{e^x}{\sqrt{1-e^x}}dx = \sin^{-1}e^x + c \quad$ (B)

22. The first three terms in the Maclaurines expansion $e^x\cos x$ is
$(A)1 + x - \frac{x^3}{2!} + --- \ (B)x + x^2 - \frac{x^3}{3!} + --- \ (C)x - \frac{x^2}{2!} - \frac{5x^4}{4!} + ---$
$(D)1 - x + \frac{x^3}{3!} + ---$

Solution

Maclaurines expansion of $e^x\cos x$ is given as

$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + ---$

$f(x) = e^x\cos x \quad f(0) = 1$

$f'(x) = e^x\cos x - e^x\sin x \quad f'(0) = 1$

$f''(x) = e^x \cos x - 2e^x \sin x - e^x \cos x \quad f''(0) = 0$
$f'''(x) = -2(e^x \sin x + e^x \cos x) \quad f'''(0) = -2$
$e^x \cos x = 1 + 1(x) + \frac{0(x^2)}{2!} + \frac{(-2)(x^3)}{3!} + - - -$
$= 1 + x + 0 - \frac{x^3}{3} + - - -$
$1 + x - \frac{x^3}{3!} + - - -$

23. If $y = (\tan^{-1} x)^2$, then $\frac{d^2y}{dx^2} + 4x\tan^{-1} x =$ (A)-1 (B) 2 (C) 1 (D) 0

Solution

Let $u = \tan^{-1} x \quad \frac{du}{dx} = \frac{1}{1+x^2}$
$y = u^2 \quad \frac{dy}{du} = 2u$
$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$\frac{dy}{dx} = 2u \times \frac{1}{1+x^2} = \frac{2\tan^{-1} x}{1+x^2}$
Second derivative
$$\frac{d^2y}{dx^2} = \frac{2(1+x^2)(\frac{1}{1+x^2}) - 2\tan^{-1} x(2x)}{(1+x^2)^2}$$
$$= \frac{2 - 4x\tan^{-1} x}{(1+x^2)^2}$$
Now $\frac{d^2y}{dx^2} + 4x\tan^{-1} x = \frac{2 - 4x\tan^{-1} x}{(1+x^2)^2} + 4x\tan^{-1} x$ No Answer

Note: the question should be

If $y = (\tan^{-1} x)^2$, then $(1+x^2)^2\frac{d^2y}{dx^2} + 4x\tan^{-1} x$ is and the answer should have been
$$(1+x^2)^2\frac{(2 - 4x\tan^{-1} x)}{(1+x^2)^2} + 4x\tan^{-1} x$$
$= 2 - 4x\tan^{-1} x + 4x\tan^{-1} x = 2$ (B)

Given the parabola $y^2 = 16x$, answer questions 24 - 26

24. The equation to the tangent at the point(9,12) is (A)$2x - 3y - 4 = 0$ (B)$3x + 2y + 10 = 0$ (C)$2x - 3y - 18 = 0$ (D)$3x - 2y + 72 = 0$

Solution

$y^2 = 16x$ differentiating implicitly
$2y\frac{dy}{dx} = 16$
$\frac{dy}{dx} = \frac{16}{2y} = \frac{8}{y}$

$\frac{dy}{dx}|_{(9,12)} = \frac{8}{12} = \frac{2}{3}$
Equation of tangent
$y - y_1 = m(x - x_1)$
$y - 12 = \frac{2}{3}(x - 9)$
$(y - 12)3 = 2(x - 9)$
$3y - 36 = 2x - 18$
$3y - 2x - 36 + 18 = 0$
$3y - 2x + 18 = 0$ No Answer

25. The angles between the tangent at the point (9,12) and (4,−8) is (A)$\frac{5\pi}{4}$ (B)$\frac{\pi}{2}$ (C)$\frac{\pi}{3}$ (D)$\frac{\pi}{4}$

Solution

$y^2 = 16x \quad \frac{dy}{dx} = \frac{8}{y}$
$m_1 = \frac{8}{12} = \frac{2}{3}$
$m_2 = \frac{8}{-8} = -1$
Angle between the tangents
$$\Longrightarrow \quad \tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$
$$\tan\theta_1 = \frac{-1 - \frac{2}{3}}{1 + (-1)(\frac{2}{3})}$$
$$= \frac{\frac{-5}{3}}{\frac{1}{3}} = \frac{-5}{3} \times \frac{3}{1} = -5$$
$$\tan\theta_2 = \frac{m_2 - m_1}{1 + m_2 m_1}$$
$$\frac{\frac{2}{3} - (-1)}{1 + (\frac{2}{3})(-1)}$$
$\theta = \tan^{-1} 5 = 78.69^0$ No Answer

26. The volume of revolution formed by rotating the part of the parabola form x=1 to x=4 about the x-axis in cubic unit is (A)5376π (B)1536π (C)256π (D)72π

Solution

$y^2 = 16x$
Revolution $= \int_{x_1}^{x_2} \pi y^2 dx$
$= \int_1^4 \pi 16x dx$
$= 16\pi \int_1^4 x dx = 16\pi[\frac{x^2}{2}]_1^4 + c$
$= \frac{16\pi}{2}[(4^2 + c) - (1^2 + c)]$

Scanned by CamScanner

$= \frac{16\pi}{2}(16-1) = 8\pi \times 15 = 120\pi$ No Answer

27. The curve passing through (2,1) whose gradient is $1+2x-3x^2$ is (A)$x+x^2-x^3-2$ (B)$x-x^2-x^3+2$ (C)$x+x^2-x^3+3$ (D)$y=x-x^2+x^3-1$

Solution

Gradient is $1+2x-3x^2$ at point (2,1)
Integration and substitute the value of x and y to get 'C' and then replace 'C' in the equation.

$\frac{dy}{dx} = 1+2x-3x^2, \quad dy = 1+2x-3x^2dx$

Integrating

$y = \int 1+2x-3x^2 dx$
$y = x+x^2-x^3+c$ at (2,1)
$1 = 2+2^2-2^3+c$
$1 = 2+4-8+c$
$1 = -2+ \quad \therefore C=3$

The curve is $y = x+x^2-x^3+3$

At time t, the velocity of a particle moving in a straight line is increasing at the rate of $(2t-\frac{4}{t^3})$. When t=1, the velocity is 6 and at that time the particle is at distance 34/3. Answer questions 28 - 30.

Let $v = 2t - \frac{4}{t^3} \quad t=1 \quad v=6$
$d = \frac{34}{3} = s$
$\frac{dv}{dt} = (2t-\frac{4}{t^3}) = a$
$v = \int adt = \int (2t-4t^{-3})dt = t^2 - \frac{4t^{-2}}{-2} + c$
$v = t^2+2t^{-2}+c$ at $t=1 \quad v=6$
$6 = 1^2+2(1)^{-2}+c$
$6 = 1+2+c \quad c=3$
$v = t^2+2t^{-2}+3 = \frac{ds}{dt} \quad ------(i)$
$s = \int (t^2+2t^{-2}+3)dt = \frac{t^3}{3} + \frac{2t^{-1}}{-1} + 3t + c$
$s = \frac{1}{3}t^3 - 2t^{-1} + 3t + c$
at $t=1 \quad s=\frac{34}{3}$
$\frac{34}{3} = \frac{1}{3} - 2 + 3 + c, \quad \frac{34}{3} = \frac{1-6+9}{3} + c$
$c = \frac{34}{3} - \frac{4}{3} = \frac{30}{3} = 10$
$s = \frac{t^3}{3} - 2t^{-1} + 3t + 10 \quad ------(ii)$

28. How far is the particle from the origin 3 seconds later? (A)$\frac{62}{3}$

(B)$\frac{61}{2}$ (C)8 (D) $\frac{64}{3}$

Solution

at $t=3$ seconds
$s = \frac{3^3}{3} - 2(3)^{-1} + 3(3) + 10$
$= 9 - \frac{2}{3} + 9 + 10$
$= \frac{27-2+27+30}{3} = \frac{82}{3}$ No Answer

29. What is the velocity after 2 seconds? (A)$\frac{20}{4}$ (B)$\frac{15}{2}$ (C)$\frac{31}{4}$ (D)$\frac{25}{2}$

Solution

$v = t^2+2t^{-2}+3$
at $t=2$ sec $\quad v = 2^2+2(-2)^{-2}+3$
$v = 4+\frac{2}{4}+3$
$= 7+\frac{1}{2} = \frac{14+1}{2} = \frac{15}{2}$
$v = 7.5m/s$ (B)

30. The equation of velocity, $v=$ (A)$t^2+2t^{-2}+3$ (B)$2t^2-t^{-1}+3$ (C)$t^2+3t^{-2}+4$ (D)$2t^2+2t^{-2}+3$

Solution

The equation of the velocity
$v = t^2+\frac{2}{t^2}+3$
$v = t^2+2t^{-2}+3$ (A)

31. If $y = \frac{\sin^{-1}2x}{\sqrt{1-4x^2}}$, simplify $\frac{dy}{dx} - 4xy$ to get (A)$4x$ (B)2 (C)0 (D)$\sin^{-1}x$

Solution

Let $u = \sin^{-1}2x \quad \frac{du}{dx} = \frac{2}{\sqrt{1-4x^2}}$
$v = \sqrt{1-4x^2} \quad \frac{dv}{dx} = \frac{-8x}{2\sqrt{1-4x^2}}$

$$\frac{dy}{dx} = \left[(\sqrt{1-4x^2})(\frac{2}{\sqrt{1-4x^2}}) + \frac{4x\sin^{-1}2x}{\sqrt{1-4x^2}}\right] \div (\sqrt{1-4x^2})^2$$

$$\frac{dy}{dx} = \left(2+\frac{4x\sin^{-1}2x}{\sqrt{1-4x^2}}\right) \div 1-4x^2$$

For $\frac{dy}{dx} - 4xy$

$$= \frac{2+\frac{4x\sin^{-1}2x}{\sqrt{1-4x^2}}}{1-4x^2} - \frac{4x\sin^{-1}2x}{\sqrt{1-4x^2}}$$

$$= \frac{2\sqrt{1-4x^2}+4x\sin^{-1}2x}{(1-4x^2)\sqrt{1-4x^2}} - \frac{4x\sin^{-1}2x}{\sqrt{1-4x^2}}$$

$$= \frac{2\sqrt{1-4x^2}+4x\sin^{-1}2x-4x\sin^{-1}2x-4x\sin^{-1}2x(1-4x^2)}{(1-4x^2)\sqrt{1-4x^2}}$$

$$= \frac{2\sqrt{1-4x^2} + 4x\sin^{-1}2x - 4x\sin^{-1}2x + 16x^3\sin^{-1}2x}{(1-4x^2)^{\frac{3}{2}}}$$

$$\frac{2\sqrt{1-4x^2} + 16x^3\sin^{-1}2x}{(1-4x^2)^{\frac{3}{2}}}$$

Note: The question should be

If $y = \dfrac{\sin^{-1}2x}{\sqrt{1-4x^2}}$, simplify $(1-4x^2)\dfrac{dy}{dx} - 4xy$

and the solution/Answer should have been

$$\frac{dy}{dx} = \frac{2 + \frac{4x\sin^{-1}2x}{\sqrt{1-4x^2}}}{(1-4x^2)}$$

Then $(1-4x^2)\left[\dfrac{2 + \frac{4x\sin^{-1}2x}{\sqrt{1-4x^2}}}{(1-4x^2)}\right] - \dfrac{4x\sin^{-1}2x}{\sqrt{1-4x^2}}$

$$= 2 + \frac{4x\sin^{-1}2x}{\sqrt{1-4x^2}} - \frac{4x\sin^{-1}2x}{\sqrt{1-4x^2}} = 2 \quad \text{(B)}$$

32. If $y = \tan^{-1}(\frac{2x}{a})$, then $(a^2+4x^2)\frac{dy}{dx} =$ (A)a^2 (B)0 (C)ax (D)$2a$

Solution

Let $u = \frac{2x}{a}$ $\quad \frac{du}{dx} = \frac{2}{a}$

$y = \tan^{-1}u$ $\quad \frac{dy}{du} = \frac{1}{1+u^2}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{1+u^2} \times \frac{2}{a}$$

$$= \frac{\frac{2}{a}}{1+(\frac{2x}{a})^2} = \frac{\frac{2}{a}}{\frac{a^2+4x^2}{a^2}}$$

$$\frac{2}{a} \times \frac{a^2}{a^2+4x^2} = \frac{2a}{a^2+4x^2}$$

Now, For $(a^2+4x^2)\dfrac{dy}{dx}$

$$\Longrightarrow \quad (a^2+4x^2)\left(\frac{2a}{a^2+4x^2}\right) = 2a \quad \text{(D)}$$

33. If $e^t\tan t$, then $e^{-t}\frac{dy}{dx} - \tan t =$ (A)$\tan t$ (B)1 (C)$\sec^2 t$ (D)$\tan^2 t$

Solution

Let $u = e^t$ $\quad \frac{du}{dt} = e^t$

$v = \tan t$ $\quad \frac{dv}{dt} = \sec^2 t$

$$\frac{dy}{dt} = U\frac{dv}{dt} + V\frac{du}{dt}$$

$= e^t\sec^2 t + e^t\tan t$

$= e^t(\sec^2 t + \tan t)$

Now, for $\quad e^{-t}\dfrac{dy}{dt} - \tan t$

$\Longrightarrow \quad e^{-t}[e^t(\sec^2 t + \tan t)] - \tan t$

$\sec^2 t + \tan t - \tan t = \sec^2 t \quad$ (C)

34. If $y = e^x + \tanh^{-1}e^{-x}$, then $\frac{1}{2}(1+e^{2x})\frac{dy}{dx} - e^{2x} =$ (A)e^{4x} (B)e^{-2x} (C)e^{2x} (D)e^x

Solution

Let $u = e^{-x}$ $\quad \frac{du}{dx} = -e^{-x}$

$v = \tanh^{-1}u$ $\quad \frac{dv}{du} = \frac{1}{1-u^2}$

$$\frac{dy}{dx} = \frac{de^x}{dx} + \frac{dv}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = e^x + \frac{1}{1-u^2} \times -e^{-x}$$

$$e^x - \frac{e^{-x}}{1-e^{-2x}} = \frac{e^x - e^{-x} - e^{-x}}{1-e^{-2x}}$$

$$\frac{dy}{dx} = \frac{e^x - 2e^{-x}}{1-e^{-2x}}$$

Then $\quad \frac{1}{2}(1+e^{2x})\frac{dy}{dx} - e^{2x}$. It is not possible, no assumption. consider it void (invalid).

35. If $y = \sinh x$, then $\dfrac{d^7y}{dx^7} =$ (A)$\cosh x$ (B)$\cosh x$ (C)$\sinh x$ (D)$-\sinh x$

Solution

$y = \sinh x$

$\frac{dy}{dx} = \cosh x$; $\quad \frac{d^2y}{dx^2} = \sinh x$

$\frac{d^3y}{dx^3} = \cosh x$; $\quad \frac{d^4y}{dx^4} = \sinh x$

$\frac{d^5y}{dx^5} = \cosh x$; $\quad \frac{d^6y}{dx^6} = \sinh x$

$\dfrac{d^7y}{dx^7} = \cosh x \quad$ (B)

36. If $y = \cos x$, then $\dfrac{d^{10}y}{dx^{10}}$ is (A)$-\sin x$ (B)$\cos x$ (C)$-\cos x$ (D)$\sin x$

Solution

$y = \cos x$ $\quad \frac{dy}{dx} = -\sin x$

$\frac{d^2y}{dx^2} = -\cos x$ $\quad \frac{d^3y}{dx^3} = \sin x$

$\frac{d^4y}{dx^4} = \cos x$ $\quad \frac{d^5y}{dx^5} = -\sin x$

$\frac{d^6y}{dx^6} = -\cos x$ $\quad \frac{d^7y}{dx^7} = \sin x$

$\frac{d^8y}{dx^8} = \cos x$ $\quad \frac{d^9y}{dx^9} = -\sin x$

$\frac{d^{10}y}{dx^{10}} = -\cos x \quad$ (C)

37. If $y = e^{2x}\cos 2x$, then $\frac{1}{2}e^{-2x}\frac{dy}{dx} + 2\sin^2 x =$ (A)$2\cos^2 x\sin^2 x$ (B)$(\cos x - \sin x)^2$ (C)$\cos^2 x + 2\sin^2 x$ (D)$(\cos x + \sin x)^2$

Solution

Let $u = e^{2x}$ $\frac{du}{dx} = 2e^{2x}$

$v = \cos 2x$ $\frac{dv}{dx} = -2\sin 2x$

$\frac{dy}{dx} = U\frac{dv}{dx} + V\frac{du}{dx}$

$= e^{2x}(-2\sin 2x) + \cos 2x(2e^{2x})$

$= -2e^{2x}\sin 2x + 2e^{2x}\cos 2x$

$2e^{2x}(\cos 2x - \sin 2x)$

Then $\frac{1}{2}e^{-2x}\frac{dy}{dx} + 2\sin^2 x$

$\frac{1}{2}e^{-2x}[2e^{2x}(\cos 2x - \sin 2x)] + 2\sin^2 x$

$= \cos 2x - \sin 2x + 2\sin^2 x$

$= \cos^2 x - \sin^2 x - 2\sin x\cos x + 2\sin^2 x$

$= \cos^2 x + \sin^2 x - 2\sin x\cos x$

$= (\cos x - \sin x)^2$ (B)

Perform the operation indicated in questions 38 - 40

38. $\int\frac{dx}{\sqrt{9-4x^2}}$ (A)$\frac{3}{2}\cos^{-1}(\frac{x}{2})+c$ (B)$\frac{1}{2}\sin^{-1}(\frac{2x}{3})+c$ (C)$\cos^{-1}(\frac{2x}{3})+c$ (D)$\frac{2}{3}\sin^{-1}(\frac{3x}{2})+c$

Solution

$\int\frac{dx}{\sqrt{9-4x^2}} = \int\frac{dx}{\sqrt{3^2(2x)^2}}$

Lets substitute $2x = 3\sin\theta$

$\theta = \sin^{-1}(\frac{2x}{3})$

$\frac{2dx}{d\theta} = 3\cos\theta,\quad dx = \frac{3}{2}\cos\theta d\theta$ - - - - - - - (1)

square both sides.

$(2x)^2 = 3^2\sin^2\theta$

$4x^2 = 9(1-\cos^2\theta)$

$4x^2 = 9 - 9\cos^2\theta$

$9\cos^2\theta = 9 - 4x^2$

$\cos^2\theta\frac{9-4x^2}{9}$

$\cos\theta = \frac{\sqrt{9-4x^2}}{3}$

$3\cos\theta = \sqrt{9-4x^2}$ - - - - - - - (2)

Now, Lets substitute equations (1) and (2)

$\int\frac{dx}{\sqrt{9-4x^2}} = \int\frac{\frac{3}{2}\cos\theta}{3\cos\theta} = \frac{1}{2}\int d\theta$

$\frac{1}{2}\int d\theta = \frac{\theta}{2} + c$ but $\theta = \sin^{-1}(\frac{2x}{3})$

$\int\frac{dx}{\sqrt{9-4x^2}} = \frac{1}{2}\sin^{-1}(\frac{2x}{3}) + c$ (B)

39. $\int_0^{\frac{\pi}{2}}(5\cos^2\theta + 3\sin^2\theta)d\theta$ (A)$\frac{5\pi}{4}$ (B)$\frac{2\pi}{3}$ (C)2π (D)$\frac{3\pi}{4}$

Solution

$\int_0^{\frac{\pi}{2}}(5\cos^2\theta + 3\sin^2\theta)d\theta$

$\int_0^{\frac{\pi}{2}}5\cos^2\theta d\theta + \int_0^{\frac{\pi}{2}}3\sin^2\theta d\theta$

Recall $\cos^2\theta = \frac{\cos 2\theta + 1}{2}$, $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Now we substitute

$5\int_0^{\frac{\pi}{2}}(\frac{\cos 2\theta + 1}{2})d\theta + 3\int_0^{\frac{\pi}{2}}(\frac{1-\cos 2\theta}{2})\theta$

$\frac{5}{2}\int_0^{\frac{\pi}{2}}\cos 2\theta d\theta + \frac{5}{2}\int_0^{\frac{\pi}{2}}d\theta + \frac{3}{2}\int_0^{\frac{\pi}{2}}d\theta - \frac{3}{2}\int_0^{\frac{\pi}{2}}\cos 2\theta d\theta$

$\frac{5}{2}\left[\frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}} + \frac{5}{2}[\theta]_0^{\frac{\pi}{2}} + \frac{3}{2}[\theta]_0^{\frac{\pi}{2}} - \frac{3}{2}\left[\frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$

$= \left[\frac{5}{4}\sin 2\theta + \frac{5}{2}\theta\right]_0^{\frac{\pi}{2}} + \left[\frac{3}{2}\theta - \frac{3}{4}\sin 2\theta\right]_0^{\frac{\pi}{2}}$

$\left[\frac{2}{4}\sin 2\theta + \frac{8}{2}\theta\right]_0^{\frac{\pi}{2}}$

$\left[\frac{1}{2}\sin 2\theta + 4\theta\right]_0^{\frac{\pi}{2}}$

$= \left[\frac{1}{2}\sin(2\times\frac{\pi}{2}) + 4\times\frac{\pi}{2}\right] - \left[\frac{1}{2}\sin 2(0) + 4(0)\right]$

$= \left[\frac{1}{2}\sin\pi + 2\pi\right] - \left[\frac{1}{2}\sin 0 + 0\right]$

Recall $\sin\pi = 0$ $\sin 0 = 0$

$= \left[\frac{1}{2}(0) + 2\pi\right] - \left[\frac{1}{2}(0) + 0\right] = 2\pi$ (C)

40. $\int_0^{\frac{\pi}{3}}4\cos 4x\cos 2x dx$ (A)$\frac{5\sqrt{3}}{8}$ (B)$\frac{2\sqrt{3}}{3}$ (C)$\frac{\sqrt{3}}{2}$ (D)$\sqrt{3}$

Solution

$\int_0^{\frac{\pi}{3}}4\cos 4x\cos 2x dx = 4\int_0^{\frac{\pi}{3}}\cos 4x\cos 2x dx$

Now recall from trig functions

$\cos p\cos q = \frac{1}{2}[\cos(p+q) + \cos(p-q)]$

$4\int_0^{\frac{\pi}{3}}[\frac{1}{2}\cos(4x+2x) + \cos(4x-2x)]dx$

$2\int_0^{\frac{\pi}{3}}(\cos 6x + \cos 2x)$

$2\int_0^{\frac{\pi}{3}}\cos 6x dx + 2\int_0^{\frac{\pi}{3}}\cos 2x dx$

$2[\frac{1}{6}\sin 6x]^{\frac{\pi}{3}}_{0} + 2[\frac{1}{2}\sin 2x]^{\frac{\pi}{3}}_{0}$

$[\frac{1}{3}\sin 6x + \sin 2x]^{\frac{\pi}{3}}_{0}$

$= [\frac{1}{3}\sin 6(\frac{\pi}{3}) + \sin 2(\frac{\pi}{3})] - [\frac{1}{3}\sin 6(0) + \sin 2(0)]$

$= \frac{1}{3}\sin 2\pi + \sin\frac{2\pi}{3} - 0 + 0$

$\frac{1}{3}\sin 2\pi + \sin\frac{2\pi}{3}$

$= 0 + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (C)

0.3 MATHS 201 2016/2017 EXAMINATION

1. Find $\int_{-2}^{2}(x^3 - x^5)dx$ (A) $\frac{1}{4}$ (B) 1 (C) 0 (D) 2 (E) 4

Solution

By Integrating

$$\left[\frac{x^4}{4} - \frac{x^6}{6}\right]_{-2}^{2} = \left[\frac{2^4}{4} - \frac{2^6}{6}\right] - \left[\frac{(-2)^4}{4} - \frac{(-2)^2}{6}\right]$$

$= 4 - \frac{64}{6} - 4 + \frac{64}{6} = 0$ (C)

2. If $y = \tan^{-1}x$, then $(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} =$ (A) -1 (B) 1 (C) 0 (D) -2 (E) 2

Solution

$y = \tan^{-1}x$ $\quad \frac{dy}{dx} = \frac{1}{1+x^2}$

Using quotient rule

$\frac{d^2y}{dx^2} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

$= \frac{(1+x^2)(0) - 1(2x)}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2}$

$(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{du}{dx}$

$= (1+x^2)\frac{2x}{(1+x^2)^2} + 2x.\frac{1}{1+x^2}$

$= \frac{-2x}{1+x^2} + \frac{2x}{1+x^2} = 0$ (C)

3. If $y = 2e^{-3x}$, then $\frac{dy}{dx}$ is (A) $-9y$ (B) $9y$ (C) $6y$ (D) $-6y$ (E) $4y$

Solution

$\frac{dy}{dx} = 2x(-3e^{-3x}) + e^{-3x}(2)$

$= -6xe^{-3x} + 2e^{-3x}$

$\frac{d^2y}{dx^2} = -6x(-3e^{-3x}) + e^{-3x}(-6) - 6e^{-3x}$

$= 18xe^{-3x} - 12e^{-3x}$

$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 18xe^{-3x} - 12e^{-3x} + 6(-6xe^{-3x} + 2e^{-3x})$

$= 18xe^{-3x} - 12e^{-3x} - 36xe^{-3x} + 12e^{-3x}$

$-18xe^{-3x} = 9(-2xe^{-3x})$

but $\quad y = 2xe^{-3x}$

$\therefore \quad = -9y \qquad$ (A)

4. Find $\int_0^4 \frac{dx}{\sqrt{16-x^2}}$ (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Solution

from standard integral

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(\frac{x}{a}) + c$

comparing

$\int_0^4 \frac{dx}{\sqrt{16-x^2}} = \left[\sin^{-1}\frac{x}{4}\right]_0^4$

$= \left[\sin^{-1}\frac{4}{4} + c\right] - \left[\sin^{-1}\frac{0}{4} + c\right]$

$\sin^{-1}1 - \sin^{-1}0 = \frac{\pi}{2}$ (C)

5. If $y = \tan^{-1}\left(\frac{\sin t}{\cos t - 1}\right)$, then $\frac{dy}{dt}$ is (A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Solution

Let $U = \frac{\sin t}{\cos t - 1}$

$y = \tan^{-1}U \quad \frac{dy}{du} = \frac{1}{1+U^2}$

from $U = \frac{\sin t}{\cos t - 1}$ using quotient rule

$\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

$\therefore \frac{du}{dt} = \frac{(\cos t - 1)(\cos t) - (\sin t)(-\sin t)}{(\cos t - 1)^2}$

$\frac{\cos^2 t - \cos t + \sin^2 t}{(\cos t - 1)^2}$

$\frac{\cos^2 t + \sin^2 t - \cos t}{(\cos t - 1)^2}$ but $\cos^2 t + \sin^2 t = 1$

$= \frac{1 - \cos t}{(\cos t - 1)^2}$

$\therefore \frac{dy}{dx} = \frac{1}{1 + (\frac{\sin t}{\cos t - 1})^2} \times \frac{1 - \cos t}{(\cos t - 1)^2}$

$\frac{dy}{dx} = \frac{1}{1 + \frac{\sin^2 t}{(\cos t - 1)^2}} \times \frac{1 - \cos t}{(\cos t - 1)^2}$

$\frac{1}{\frac{(\cos t - 1)^2 + \sin^2 t}{(\cos t - 1)^2}} \times \frac{1 - \cos t}{(\cos t - 1)^2}$

$= \frac{(\cos t - 1)^2}{(\cos t - 1)^2} + \sin^2 t \times \frac{1 - \cos t}{(\cos t - 1)^2}$

$= \frac{1 - \cos t}{\cos^2 t} - 2\cos t + 1 + \sin^2 t = \frac{1 - \cos t}{\cos^2 t} + \sin^2 t - 2\cos t + 1$

$\frac{1 - \cos t}{1 + 1 - 2\cos t} = \frac{1 - \cos t}{2 - 2\cos t}$

$= \frac{1 - \cos t}{2(1 - \cos t)} = \frac{1}{2}$ (B)

6. The equation to the curve passing through (0,1) whose gradient is $1 - 3x^2$ is (A) $y = x - \frac{2}{3}x^3 + 2$ (B) $y = x - \frac{2}{3}x^3 - 1$ (C) $y = x + \frac{2}{3}x^3 + 2$ (D) $y = x - \frac{2}{3}x^3 + 1$ (E) $y = x - \frac{2}{3}x^3 + 2$

Solution

Integrate and substitute the value of x and y to get c and then replace c in the equation

$\frac{dy}{dx} = 1 - 3x^2$

$dy = (1 - 3x^2)dx$

Integrate both side

$\int dy = \int(1 - 3x^2)dx$

$y = x - \frac{3x^{2+1}}{2+1} + c$

$y = x - \frac{3x^3}{3} + c$

$y = x - x^3 + c$

Substitute the value of x and y (0,1)

$1 = 0 - 0 + c$

$\therefore \quad C = 1$

$\therefore \quad y = x - x^3 + 1$ (No Answer)

7. If $y = e^{3x}$, then $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} =$ (A) 1 (B) $3e^{3x}$ (C) $9e^{3x}$ (D) 0 (E) e^{3x}

Solution

$\frac{dy}{dx} = 3e^{3x} \quad \frac{d^2y}{dx^2} = 9e^{3x}$

then $\frac{d^2y}{dx^2} - 3\frac{dy}{dx}$ will be

$= 9e^{3x} - 3(3e^{3x})$

$= 9e^{3x} - 9e^{3x} = 0$ (D)

8. Find $\int \sin^2 x \cos^5 x dx$ (A) $\frac{1}{7}\sin^7 x + \frac{2}{5}\sin^5 x + \frac{1}{3}\sin^3 x + C$ (B) $\frac{1}{7}\sin^7 x - \frac{2}{5}\sin^5 x + \frac{1}{3}\sin^3 x + C$ (C) $\frac{1}{7}\sin^7 x + \frac{2}{5}\sin^5 x -$ $\frac{1}{3}\sin^3 x + C$

(D) $-\frac{1}{7}\sin^7 x + \frac{2}{5}\sin^5 x + \frac{1}{3}\sin^3 x + C$ (E) None

Solution

$\int \sin^2 x\cos^5 x dx = \int \sin^2 x(\cos^4 x)\cos x dx$

$\int \sin^2 x(1-\sin^2 x)^2\cos x dx$

$\int \sin^2 x\cos x(1-2\sin^2 x+\sin^4 x)dx$

$= \int(\sin^2 x\cos x - 2\sin^4 x\cos x + \sin^6 x\cos x)dx$

integrating

$= \frac{\sin^3 x}{3} - \frac{2}{5}\sin^5 x + \frac{\sin^7 x}{7} + C$

$= \frac{1}{7}\sin^7 x - \frac{2}{5}\sin^5 x + \frac{1}{3}\sin^3 x + C$ (B)

9. Find $\int e^{2x}\cos 3x dx$ (A) $\frac{e^{2x}}{9}(3\sin 3x + 2\cos 3x) + c$ (B) $\frac{e^{2x}}{9}(3\sin 3x - 2\cos 3x) + c$ (C) $\frac{e^{2x}}{13}(2\sin 3x + 2\cos 3x) + c$ (D) $-\frac{e^{2x}}{13}(3\sin 3x + 2\cos 3x) + c$ (E) $\frac{e^{2x}}{13}(3\sin 3x + 2\cos 3x) + c$

Solution

from standard integral

$\int e^{ax}\cos bx = \frac{e^{ax}}{a^2+b^2}(b\sin bx + a\cos bx)$

$a = 2 \quad b = 3$

comparing

$\int e^{2x}\cos 3x dx = \frac{e^{2x}}{2^2+3^2}(3\sin 3x + 2\cos 3x)$

$= \frac{e^{2x}}{13}(3\sin 3x + 2\cos 3x) + c$ (E)

10. Find $\frac{dy}{dx}$ of $y^2 - \cos 2x = 7$ at point $(\frac{\pi}{4}, -1)$ (A) $\frac{\pi}{4}$ (B) -1 (C) 0 (D) π (E) none

Solution

differentiating implicitly

$2y\frac{dy}{dx} - (-2\sin 2x) = 0$

$2y\frac{dy}{dx} + 2\sin 2x = 0$

$\frac{dy}{dx} = -\frac{2\sin 2x}{2y}|_{\frac{\pi}{4}, -1}$

$\frac{dy}{dx} = \frac{-2\sin 2(-1)}{2(\frac{\pi}{4})}$

$= \frac{2\sin 2}{90} = \frac{0.0698}{90} = 0.00077 = 0$ (C)

11. If $y = \tanh^{-1}\left(\frac{1-x}{1+x}\right)$, then $2x\frac{dy}{dx}$ is (A) 1 (B) -1 (C) $\frac{-1}{2}$ (D) $\frac{1}{x}$ (E) None

Solution

Let $U = \frac{1-x}{1+x}$

$y = \tanh^{-1}(U)$ using quotient rule

$\frac{dy}{du} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

$\frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$

$= \frac{-1-x-(1-x)}{(1+x)^2}$

$\frac{dy}{dx} = \frac{-1-x-1+x}{(1+x)^2} = \frac{-2}{(1+x)^2}$

$= \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{1-U^2} \times \frac{-2}{(1+x)^2}$

$\frac{dy}{dx} = \frac{(1+x)^2}{(1+x)^2-(1-x)^2} \times \frac{-2}{(1+x)^2}$

$\frac{-2}{(1+x)^2-(1-x)^2} = \frac{-2}{4x} = \frac{-1}{2x}$

$\frac{dy}{dx} = \frac{-1}{2x}$

from the condition

$2x\frac{dy}{dx} = 2x \times \frac{-1}{2x} = -1$ (B)

12. Find the equation of normal to the parabola if $x^2 - y^2 = 7$ at the point $(4, -3)$ (A) $3x - 4y = 24$ (B) $x - 4y = 24$ (C) $3x + 4y = 24$ (D) $3x - 4y = 7$ (E) None

Solution

differentiating implicitly

$2x - 2y\frac{dy}{dx} = 0$

$-2y\frac{dy}{dx} = -2x \implies \frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}|_{(4,-3)}$

$\frac{dy}{dx} = \frac{4}{-3}(M)$

Equating of normal $\Longrightarrow (y - y_1) = m(x - x_1)$

$Normal = m_2 = \frac{-1}{m_1} = \frac{-1}{\frac{4}{-3}} = -1 \times \frac{-3}{4} = \frac{3}{4}$

Equation of normal

$y + 3 = \frac{3}{4}(x - 4)$

$4y + 12 = 3x - 12$

$12 + 12 = 3x - 4y$

$24 = 3x - 4y$

$3x - 4y = 24$ (A)

13. If $y = ln \sec x$, then $\cos x \frac{dy}{dx}$ is (A) $\tan x$ (B) $\sec x$ (C) $\cot x$ (D) $cosec x$ (E) $\sin x$

Solution

Let $U = \sec x \quad \frac{du}{dx} = \sec x \tan x$

$y = lnU \quad \frac{dy}{du} = \frac{1}{U}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{U} \times \sec x \tan x$

$= \frac{1}{\sec x} \times \sec x \tan x$

$\frac{dy}{dx} = \tan x$

from the condition

$cosec x \frac{dy}{dx} = ?$

$\frac{dy}{dx} = \tan x = \frac{\sin x}{\cos x}$ divide both side by $\sin x$

$\frac{1}{\sin x}\frac{dy}{dx} = \frac{1}{\cos x}$

Remember $\frac{1}{\sin x} = cosec x$ and $\frac{1}{\cos x} = \sec x$

$\therefore \quad cosec x \frac{dy}{dx} = \sec x$ (B)

14. Find $\int_0^{\frac{\pi}{2}} 2\sin^2 x dx$ (A) $\frac{3\pi}{2}$ (B) $\frac{5\pi}{2}$ (C) 1 (D) $\frac{2\pi}{4}$ (E) None

Solution

from trigonometry

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$2\int \frac{1}{2}(1 - \cos 2x)dx$

$2 \times \frac{1}{2}\int 1 - \cos 2x dx$

$\int_0^{\frac{\pi}{2}} 1 - \cos 2x = \left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$

$= \left[\frac{\pi}{2} - \frac{1}{2}\sin 2\frac{\pi}{2}\right] - \left[0 - \frac{1}{2}\sin 2(0)\right]$

$= \frac{\pi}{2} - \frac{1}{2}\sin \pi = \frac{\pi}{2}$ (E)

15. If the Maclaurins series expansion of $e^{\frac{x}{2}}$ is $a + bx + cx^2 + dx^3 + ---$ then the value of d is (A) $\frac{1}{8}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{48}$ (D) $-\frac{1}{48}$

Solution

Maclaurins series

$= f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4}f^{iv}(0) + ----$

$f(x) = e^{\frac{x}{2}}$

$f'(x) = \frac{e^{\frac{x}{2}}}{2} = f'(0) = \frac{1}{2}$

$f''(x) = \frac{e^{\frac{x}{2}}}{4} = f''(0) = \frac{1}{4}$

$f'''(x) = \frac{e^{\frac{x}{2}}}{8} = f'''(0) = \frac{1}{8}$

$f^{iv}(x) = \frac{e^{\frac{x}{2}}}{16} = f^{iv}(0) = \frac{1}{16}$

By comparison

$dx^3 = \frac{x^3}{3!} \times \frac{1}{8}$

$d = \frac{1}{3 \times 2 \times 8} = \frac{1}{48}$ (C)

16. Evaluate $\int_0^{\frac{\pi}{2}} x\cos(x)dx$ (A) $\frac{\pi}{2}$ (B) 0 (C) $\frac{\pi}{2} - 1$ (D) 1

Solution

Using integral by part

$u = x \quad du = dx$

$dv = \cos x \quad v = \sin x$

$= uv - \int vdu$

$= x\sin - \int \sin x$

$= x\sin x - (-\cos x) = x\sin x + \cos x|_0^{\frac{\pi}{2}}$

$\left[\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right] - [0\sin 0 + \cos 0]$

$\frac{\pi}{2} \times 1 + 0 - 0 - 1$

$= \frac{\pi}{2} + 0 - 0 - 1 = \frac{\pi}{2} - 1$ (C)

17. If $y = \tan^{-1}\frac{x}{2}$, then $(4+x^2)\frac{dy}{dx} =$ (A) 1 (B) 0 (C) x (D) 2

Solution

$y = \tan^{-1}U \quad U = \frac{x}{2}$

$\frac{du}{dx} = \frac{2(1) - x(0)}{2^2} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{dy}{du} = \frac{1}{1+U^2}$

$= \frac{1}{1+(\frac{x}{2})^2} = \frac{1}{\frac{1}{1}+\frac{x^2}{4}} = \frac{1}{\frac{4+x^2}{4}} = \frac{4}{4+x^2}$

$\frac{dy}{dx} = \frac{4}{4+x^2} \times \frac{1}{2} = \frac{2}{4+x^2}$

then $(4+x^2)\frac{dy}{dx}$

$= 4 + x^2 \times \frac{2}{4+x^2} = 2$ (D)

18. If $y = sech^{-1}(\cos x)$, then $\cos x\frac{dy}{dx}$ is (A) 1 (B) $\sec x$ (C) $-cosecx$ (D) -1

Solution

Let $u = \cos x \quad \frac{du}{dx} = -\sin x$

$y = sech^{-1}u$

$\frac{dy}{du} = \frac{-1}{u\sqrt{1-u^2}}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{-1}{u\sqrt{1-u^2}} \times -\sin x$

$= \frac{\sin x}{\cos x\sqrt{1-cos^2 x}}$

Remember $\sin^2 x + \cos^2 x = 1$

$1 - \cos^2 x = \sin^2 x$

$= \frac{\sin x}{\cos x\sqrt{\sin^2 x}} = \frac{\sin x}{\cos x \sin x}$

$= \frac{1}{\cos x} = \sec x$ (B)

19 Evaluate $\int_0^\infty x^3 e^{\frac{-1}{2}x}dx$ (A) 4 (B) 16 (C) ∞ (D) 0 (E) none

Solution

Let $u = x^3 \quad du = 3x^2$

$dv = e^{\frac{-1}{2}x} \quad v = -2e^{\frac{-1}{2}x}$

Using integral by part

$uv - \int vdu$

$x^3. - 2e^{\frac{-1}{2}x} - \int -2e^{\frac{-1}{2}x}.3x^2$

$= -2x^3e^{\frac{-1}{2}x} - \int -2e^{\frac{-1}{2}x}.3x^2$

$= -2x^3e^{\frac{-1}{2}x} + 6\int x^2e^{\frac{-1}{2}x}$

Integrating again

$\int x^2e^{\frac{-1}{2}x}$

Let $u = x^2 \quad du = 2x$

$dv = e^{\frac{-1}{2}x} \quad dv = -2e^{\frac{-1}{2}x}$

substituting now

$-2x^2e^{\frac{-1}{2}x} - \int -2e^{\frac{-1}{2}x}.2x$

$-2x^2e^{\frac{-1}{2}x} + 4\int xe^{\frac{-1}{2}x}$

Integrating again

$\int xe^{\frac{-1}{2}x}$

$u = x \quad du = 1$

$dv = e^{\frac{-1}{2}x} \quad v = -2e^{\frac{-1}{2}x}$

$x. - 2e^{\frac{-1}{2}x} - \int -2e^{\frac{-1}{2}x}.1$

$-2xe^{\frac{-1}{2}x} + 2\int e^{\frac{-1}{2}x}$

$-2xe^{\frac{-1}{2}x} + 2[-2e^{\frac{-1}{2}x}]$

$-2xe^{\frac{-1}{2}x} - 4e^{\frac{-1}{2}x}$

Back to the equation

$\int_0^\infty x^3e^{\frac{-1}{2}x}dx = -2x^3e^{\frac{-1}{2}x}+6(-2x^2e^{\frac{-1}{2}x})+4(-2xe^{\frac{-1}{2}x}-4e^{\frac{-1}{2}x})|_0^\infty$

$= -2x^3e^{\frac{-1}{2}x} - 12x^2e^{\frac{-1}{2}x} - 8xe^{\frac{-1}{2}x} - 16e^{\frac{-1}{2}x}|_0^\infty$

$= e^{\frac{-1}{2}x}(-2x^3 - 12x^2 - 8x - 16)|$

$= e^{\frac{-1}{2}\infty}(-2(\infty)^3 - 12(\infty)^2 - 8(\infty) - 16) - e^{\frac{-1}{2}(0)}(-2(0)^3 - 12(0)^2 - 8(0) - 16)$

$= e^{-\infty}(-\infty - \infty - \infty - 16) - (-16)$

$\frac{1}{e^\infty}(-3\infty - 16) + 16 = 0 + 16 = 16$ (E)

20. Find $\int \frac{dx}{9-x^2}$ (A) $\frac{5}{6}ln(\frac{3-x}{3+x})$+c (B) $\frac{1}{6}ln(\frac{3-x}{3+x})$+c (C) $\frac{1}{6}ln(\frac{3+x}{3-x})$+c (D) $\frac{5}{6}ln(\frac{3-x}{3+x})$ + c (E) None

Solution

From standard integral

$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}ln\frac{x+a}{x-a} + c$

$a^2 = 9 \implies a = \sqrt{9} = 3$

$\int \frac{dx}{9-x^2} = \frac{1}{2\times 3}ln\frac{x+3}{x-3} + c$

$= \frac{1}{6}ln\frac{x+3}{x-3} + c$ (E)

In case there is **No 'none of the above'** among the option, you can choose option 'C' $(\frac{1}{6}ln\frac{3+x}{3-x})$

21. Let $f(x) = \left(\frac{1+x}{1-x}\right)$, then the 3^{rd} derivative of $f(x)$ is (A) $\frac{12}{(1-x)^3}$ (B) $\frac{-12}{(1-x)^3}$ (C) $\frac{12}{(x-1)^4}$ (D) $\frac{12}{(1-x)^4}$ (E) $\frac{-12}{(1-x)^4}$

Solution

$f(x) = \left(\frac{1+x}{1-x}\right)$, then the 3^{rd} derivative of f(x) is

$f(x) = \frac{1+x}{1-x}$ then $\frac{dy}{dx} = f'(x) = \frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}$

$= \frac{1-x-(-1-x0}{(1-x)^2} = \frac{1-x+1+x}{(1-x)^2} = \frac{2}{(1-x)^2}$

$\frac{d^2y}{dx^2} = f''(x) = 2(1-x)^{-2} = -4(1-x)^{-3} \times -1 = 4(1-x)^3$

$\frac{d^3y}{dx^3} = f'''(x) = -12(1-x)^{-4} \times -1 = \frac{12}{(1-x)^4}$ (D)

22. If $x^2 - xy + y^2 = 3$, find $\frac{dy}{dx}$ at point (1,1). (A) 0 (B) −1 (C) $\propto$ (D) 1 (E) none

Solution

Differentiating implicitly

$2x - y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$(2y-x)\frac{dy}{dx} = -2x + y$

$\frac{dy}{dx} = \frac{-2x+y}{2y-x}|_{1,1} = \frac{-2(1)+1}{2(1)-1}$

$= \frac{-2+1}{2-1} = \frac{-1}{1} = -1$ (B)

23. Evaluate $\int_0^{\frac{\pi}{2}}(2\sin^3 x + 3\sin^3 x)dx$ (A) $\frac{5\pi}{4}$ (B) $\frac{3\pi}{2}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$ (E) $\frac{\pi}{6}$

Solution

$\int_0^{\frac{\pi}{2}}(2\sin^3 x + 3\sin^3 x)dx = \int_0^{\frac{\pi}{2}} 2\sin^3 x dx + \int_0^{\frac{\pi}{2}} 3\sin^3 x dx$

Picking the first one

$\int_0^{\frac{\pi}{2}} 2\sin^3 x dx$

Since $\sin^2 x = 1 - \cos^2 x$

$\int_0^{\frac{\pi}{2}} 2\sin^3 x dx = 2\int_0^{\frac{\pi}{2}} \sin x(\sin^2 x)$

$= 2\int \sin x(1-\cos^2 x)dx$

$= 2\int(\sin x - \sin x\cos^2 x)dx$

$2[-\cos x - \frac{\cos^3 x}{3}]$

$= -2\cos x - \frac{2\cos^3 x}{3} + c$

Repeat the same procedure for $\int \sin^3 x dx$

$\int \sin^3 x dx = -3\cos x - \frac{3\cos^3 x}{3} + c$

$\therefore$ combine the answers together

$\int_0^{\frac{\pi}{2}}(2\sin^3 x + 3\sin^3 x)dx = -2\cos x - \frac{2\cos^3 x}{3} - 3\cos x - \frac{3\cos^3 x}{3} + c\,|_0^{\frac{\pi}{2}}$

Substituting the upper and lower limits

$[-2\cos(\frac{\pi}{2}) - \frac{2\cos(\frac{\pi}{2})^3}{3} - 3\cos(\frac{\pi}{2}) - \frac{3\cos(\frac{\pi}{2})^3}{3}] - [-2\cos(0) - \frac{2\cos(0)^3}{3} - 3\cos(0) - \frac{3\cos(0)^3}{3}]$

$(0-2-0-1)-(-2-2-3-1) = -3+8 = 5$ No Answer

24. If $x = 2\sin t$ and $y = 3\cos 2t$, then $\frac{d^2y}{dx^2}$ is (A) −3 (B) 3 (C) −2 (D) 6 (E) −6

Solution

$x = 2\sin t \quad \frac{dx}{dt} = 2\cos t$

$y = 3\cos 2t \quad \frac{dy}{dt} = -6\sin 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-6\sin 2t}{2\cos t}$

$= \frac{-6(2\sin t\cos t)}{2\cos t} = -6\sin t$

$\frac{d^2y}{dx^2} = \frac{d}{dt}(-6\sin t)\frac{dt}{dx} = \frac{-6\cos t}{2\cos t} = -3$ (A)

25. Evaluate $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos x dx$ (A) 1 (B) 2 (C) ∞ (D) 0 (E) none

Solution

By Integrating

$[\sin x]_{\frac{-\pi}{2}}^{\frac{\pi}{2}} + c = [\sin\frac{\pi}{2}] - [\sin(\frac{-\pi}{2})]$

$1-(-1) = 1+1 = 2$ (B)

26. The volume of revolution formed by rotating the part of the parabola from $x = 1$ to $x = 4$ about the x-axis in a cube unit is (A) 540π (B) 120π (C) 1080π (D) 72π (E) none

Solution

$y^2 = 16x$

Revolution about the x-axis

$= \int_{x_1}^{x_2} \pi y^2 dx$

$= \int_1^4 \pi 16 x dx = 16\pi \int_1^4 x dx$

$16\pi \int_1^4 x dx = 16\pi [\frac{x^2}{2}]_1^4 + c$

$\frac{16\pi}{2}[4^2 - 1^2] = \frac{16\pi}{2}[16 - 1]$

$= 8\pi \times 15 = 120\pi$ (B)

27 If $y = \cosh x$, then $\frac{d^{10}y}{dx^{10}}$ is (A) $\cosh x$ (B) $-\cosh x$ (C) $-\sinh x$ (D) $-\sinh x$ (E) none

Solution

$y = \cosh x \quad \frac{dy}{dx} = \sinh x$

$\frac{d^2y}{dx^2} = \cosh x \quad \frac{d^3y}{dx^3} \sinh x$

$\frac{d^4y}{dx^4} = \cosh x \quad \frac{d^5y}{dx^5} \sinh x$

$\frac{d^6y}{dx^6} = \cosh x \quad \frac{d^7y}{dx^7} \sinh x$

$\frac{d^8y}{dx^8} = \cosh x \quad \frac{d^9y}{dx^9} \sinh x \quad \frac{d^{10}y}{dx^{10}} = \cosh x$ (A)

28 Find $\int \frac{dx}{9-5x}$. (A) $\tan^{-1} 5x + c$ (B) $-5\log(9 - 5x) + c$ (C) $\log(9 - 5x) + c$ (D) $-\frac{1}{5}\log(9 - 5x) + c$ (E) none

Solution

Let $u = 9 - 5x \quad \frac{du}{dx} = -5$

$du = -5dx \quad dx = \frac{du}{-5}$

$\int \frac{1}{u} \times \frac{du}{-5}$

$= \frac{-1}{5} \int \frac{du}{u} = -\frac{1}{5} ln u$

$= \frac{-1}{5} ln(9 - 5x) + c$ (D)

29. If $y = \sin x$, then $\frac{d^9y}{dx^9}$ is (A) $-\cos x$ (B) $\cos x$ (C) $-\sin x$ (D) $\sin x$ (E) *none*

Solution

$y = \sin x$

$\frac{dy}{dx} = \cos x \quad \frac{d^2y}{dx^2} = -\sin x$

$\frac{d^3y}{dx^3} = -\cos x \quad \frac{d^4y}{dx^4} = \sin x$

$\frac{d^5y}{dx^5} = \cos x \quad \frac{d^6y}{dx^6} = -\sin x$

$\frac{d^7y}{dx^7} = -\cos x \quad \frac{d^8y}{dx^8} = \sin x$

$\frac{d^9y}{dx^9} = \cos x$ (B)

30. Find $\int \frac{4dx}{(1+x)^2(1-x)}$ (A) $ln(\frac{1+x}{1-x}) + c$ (B) $ln(\frac{1+x}{1-x}) + \frac{2}{x+1} + c$ (C) $ln(\frac{1+x}{1-x}) - \frac{1}{x+1} + c$ (D) $ln(\frac{1+x}{1-x}) + \frac{1}{x+1} + c$ (E) none

Solution

Integrating by partial fraction

$$\frac{4}{(1+x)^2(1-x)} = \frac{A}{1+x^2} + \frac{B}{(1+x)^2} + \frac{C}{1-x}$$

$$\frac{4}{(1+x)^2(1-x)} = \frac{A(1+x)(1-x) + B(1-x) + C(1+x)^2}{(1+x)^2}(1-x)$$

$4 = A(1+x)(1-x) + B(1-x) + C(1+x)^2$

When $x = 1$; $A(0) + B(0) + C(1+1)^2$

When $x = -1$; $A(0) + B(1+1) + C(0) = 4$

$2B = 4$ *then* $B = 2$

When $x = 0$

$A(1+0)(1-0) + B(1-0) + c(1+0)^2 = 4$

$A + B + C = 4$

$A = 4 - 3 = 1$

$$\therefore \int \frac{4}{(1+x)^2(1-x)}dx = \int \frac{1}{1+x}dx + \int \frac{2}{(1+x)^2}dx + \int \frac{1}{1-x}dx$$

Solving

$\int \frac{2}{(1+x)^2}dx \quad$ Let $u = 1 + x \quad du = dx$

$2\int \frac{1}{u^2} = 2\int u^{-2} = -2(\frac{1}{u}) = \frac{-2}{1+x}$

Generally, Now

$$\int \frac{4}{(1+x)^2(1-x)}dx = ln(1+x) - ln(1-x) - \frac{2}{1+x}$$

$$= ln\left(\frac{1+x}{1-x}\right) - \frac{2}{x+1} + C \quad \text{(E)}$$

0.4 MATHS 207 2015/2016 EXAMINATION

1. A matrix with $a_{ij} = 0$ whenever $i < j$ is called ? (a) Identity matrix (b) Upper triangle matrix (c) Null matrix (d) Lower triangle matrix (e) None

Solution

D (Lower triangular matrix)

2. If in a Matrix A in which $R_j = R_k$, then $|A|$ is (a) 1 (b) 2 (c) 0 (d) k (e) $\frac{1}{k}$

Solution

C (0)

Let $A = (a_{ij}) = \begin{pmatrix} 1 & 3 & 6 & 2 \\ 4 & 0 & -2 & 4 \\ 2 & 7 & 6 & -1 \\ 0 & 1 & 5 & 3 \end{pmatrix}$, if $B = (b_{ij})$ and $C = (c_{ij})$ are symmetric anti (skew) symmetric matrices respectively, such as $A = B + C$, then use the information to answer the next five questions that follows

3. Which of the following is not true? (a) $B^T = B$ (b) $C^T = -C$ (c) $2B = A^T + A$ (d) $C^T = C$ (e) $2C = A - A^T$

Solution

D $(C^T = C)$

4. b_{22} is ? (a) $\frac{7}{2}$ (b) 0 (c) $\frac{5}{2}$ (d) 1 (e) 2

Solution

Since B is symmetric

$$B = \frac{A^T + A}{2}$$

$$A = \begin{bmatrix} 1 & 3 & 6 & 2 \\ 4 & 0 & -2 & 4 \\ 2 & 7 & 6 & -1 \\ 0 & 1 & 5 & 3 \end{bmatrix} \implies A^T = \begin{bmatrix} 1 & 4 & 2 & 0 \\ 3 & 0 & 7 & 1 \\ 6 & -2 & 6 & 5 \\ 2 & 4 & -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & \frac{7}{2} & 4 & 1 \\ \frac{7}{2} & 0 & \frac{5}{2} & \frac{5}{2} \\ 4 & \frac{5}{2} & 6 & 2 \\ 1 & \frac{5}{2} & 2 & 3 \end{bmatrix} \quad \therefore \ b_{22} = 0 \quad \text{(B)}$$

5. $b_{13} + c_{31}$ is ? (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

Solution

Since C is anti - symmetrical

$C = \frac{1}{2}(A - A^T)$

$$C = \begin{bmatrix} 0 & \frac{-1}{2} & 2 & 1 \\ \frac{1}{2} & 0 & \frac{-9}{2} & \frac{3}{2} \\ -2 & \frac{9}{2} & 0 & -3 \\ -1 & \frac{-3}{2} & 3 & 0 \end{bmatrix} \quad b_{13} + b_{31} = 4 - 2 = 2$$

(B)

6. If $D = (d_{ij})$ is such that $B + D = D + B = I_4$, then d_{22} is ? (a) 0 (b) 1 (c) -1 (d) 2 (e) -2

Solution

$B + D = D + B = I_4$

$\therefore \ D = I_4 - B$

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & \frac{-7}{2} & -4 & -1 \\ \frac{-7}{2} & 1 & \frac{-5}{2} & \frac{-5}{2} \\ -4 & \frac{-5}{2} & -5 & -2 \\ -1 & \frac{-5}{2} & -2 & -2 \end{bmatrix} \quad d_{22} = 1 \quad \text{(B)}$$

7. Suppose $E = (e_{ij})$ is such that, $E + C = C + E = 0$. then e_{43} is ? (a) -2 (b) 2 (c) 3 (d) -3 (e) 1

Solution

$E + C = C + E = 0$

$E = 0 - C$

$$E = \begin{bmatrix} 0 & \frac{1}{2} & -2 & -1 \\ \frac{-1}{2} & 0 & \frac{9}{2} & \frac{-3}{2} \\ 2 & \frac{-9}{2} & 0 & 3 \\ 1 & \frac{3}{2} & -3 & 0 \end{bmatrix} \quad e_{43} = -3 \quad \text{(D)}$$

8. A matrix **A** is such that $A^2 = A$ is called (a) Idempotent (b) Symmetric (c) Triangular (d) Scalar (e) Identity
Solution
A (Idempotent)

9. Let $A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 6 & 2 & 0 \end{pmatrix}$ then $|A|$ is ? (a) 1 (b) 2 (c) -1 (d) 0 (e) 4
Solution
D (0)

10. The system $AX = 0$ will always have (a) Infinite solution (b) No solution (c) At least one solution (d) only one solution (e) None
Solution
(D) only one solution

11. A matrix **B** of order n with the property that, another matrix **A** of order n, $AB = BA = A$ is called? (a) Singular matrix (b) Inverse of A (c) Null matrix (d) Square matrix (e) Identity of A
Solution
E (Identity of A)

12. If **A** is symmetric, then (a) $A = -A^2$ (b) $A = A^T$ (c) $A = -A^T$ (d) $A = A^2$ (e) $A = -A$
Solution
B $A = A^T$

13. The inverse of **ABC** is? (a) $A^{-1}B^{-1}C^{-1}$ (b) $C^{-1}B^{-1}A^{-1}$ (c) $A^{-1}C^{-1}B^{-1}$ (d) $C^{-1}A^{-1}B^{-1}$ (e) $B^{-1}C^{-1}A^{-1}$
Solution
$E = B^{-1}C^{-1}A^{-1}$

Suppose $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then use it to answer the following questions

14. $|B^{-1}|$ is? (a) 2 (b) 3 (c) 4 (d) 5 (e) 6
Solution
$|B^{-1}|$ The minor of B is

$$B_m = \begin{pmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{5} \end{pmatrix} \qquad |B| = \frac{1}{5}$$

$$B^{-1} = \frac{Adj\,B}{|B|} = \frac{1}{5}\begin{pmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{5} \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$|B^{-1}| = 5$ (D)

15. If $A = (a_{ij}) = B - B^{-1}$, then a_{22} is? (a) $\frac{5}{24}$ (b) $\frac{-24}{5}$ (c) $\frac{24}{5}$ (d) $\frac{5}{24}$ (e) None
Solution

$$A = \begin{pmatrix} \frac{-24}{5} & 0 & 0 \\ 0 & \frac{-24}{5} & 0 \\ 0 & 0 & \frac{-24}{5} \end{pmatrix} \qquad a_{22} = \frac{-24}{5} \qquad \text{(B)}$$

16. If **B** is obtained from **A** by performing the operation $R_j \longleftrightarrow R_i$ on **A**, then (a) $|B| = |A|$ (b) $|B| = k|A|$ (c) $|B| = \frac{1}{k}|A|$ (d) $|A| = |B| = 0$ (e) $|B| = -|A|$
Solution
E ($|B| = -|A|$)

17. Suppose **A** is of order n and the row reduced echeleon form of **A** has **r** non zero rows, then the rank of **A** is? (a) n (b) r (c) $n - r$ (d) $r - n$ (e) None
Solution
B (r)

Consider $A = \left(\begin{array}{ccc|c} 1 & 4 & 6 & k_1 \\ 0 & 1 & 3 & k_2 \\ 0 & 0 & \theta - 4 & k_3 \end{array}\right)$ use matrix **A** to answer the following three questions

18. If $\theta = 4$ and $k_3 \neq 0$, then the system represented by **A** has (a) Infinite solution (b) many solution (c) Single solution (d) N[illegible] solution (e) Two solutions
Solution
D (No solution)

19. For what value of θ and k_3 would the system has infinite soluti[illegible] (a) $\theta = 4$ $k_3 = 0$ (b) $\theta = -4$, $k_3 = 0$ (c) $\theta = 4$, $k_3 = 4$ [illegible] $\theta = \infty$, $k_3 = 0$ (e) $\theta = k_3$

Solution

$\theta = 4,\ k_3 = 0$ (A)

20. By letting $\theta = 1$ and solving the system, the value of $x_2 + 3x_3$ is? (a) k_3 (b) $k_2 - k_3$ (c) $-k_2$ (d) $k_3 - k_2$ (e) k_2

Solution

$$A = \left(\begin{array}{ccc|c} 1 & 4 & 6 & k_1 \\ 0 & 1 & 3 & k_2 \\ 0 & 0 & -3 & k_3 \end{array}\right)$$

$-3x_3 = k_3, \quad x_3 = \frac{-k_3}{3}$

$x_2 + 3x_3 = k_2$

$x_2 + 3(\frac{-k_3}{3}) = k_2$

$x_2 - k_3 = k_2, \quad x_2 = k_2 + k_3$

$x_2 + k_3 \implies k_2 + k_3 + 3(\frac{-k_3}{3}) = k_2$

$k_2 + k_3 - k_3 = k_2$ (E)

21. A matrix obtained from I_n by performing a single row (column) is called ? (a) Row matrix (b) Elementary matrix (c) Singular matrix (d) Identity matrix (e) Row Identity

Solution

Elementary matrix (B)

Assume **A,B,C** are respectively $m \times n, n \times k$ *and* $k \times l$ matrices, then use the information to answer the following two questions

22. Which of the following operation is not possible (a) AB (b) BC (c) $A + B$ *if* $m = n = k$ (d) AC (e) All are possible

Solution

$A = m \times n$

$B = n \times k$

$C = k \times l$

A C (D)

23. Suppose $m = k$, then which of the following is true (a) AB, BA (b) AB, AC (c) BC, CB (d) BC, BA (e) AB, CB

Solution

$m = k$

$AB,\ BA$ (A)

24. A matrix in which a_{ij} are equal whenever $i = j$ and $a_{ij} = 0$ whenever $i \neq j$ is called (a) Singular matrix (b) Identity matrix (c) Null matrix (d) Square matrix (e) scalar matrix

Solution

Null matrix (C)

25. A matrix **A** is inverse of **B** if (a) $A - B = I_n$ (b) $AB = A$ (c) $BA = AB = 0$ (d) $A - B = 0$ (e) $AB = BA = I_n$

Solution

$AB = BA = I_n$ (E)

Use
$$\begin{array}{rcrcrcl} x & + & y & + & z & = & 3 \\ 2x & - & y & + & 3z & = & 5 \\ x & - & 3y & + & 4z & = & 6 \end{array}$$
to answer the three questions that follow.

26. Using cramer's rule, the value of **z** is (a) $\frac{5}{13}$ (b) $\frac{-5}{13}$ (c) $\frac{-13}{5}$ (d) $\frac{13}{5}$ (e) 13

Solution

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 3 \\ 1 & -3 & 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$$

$|A| = 1(-4 + 9) - 1(8 - 3) + 1(-6 + 1) = 5 - 5 - 5$

$|A| = -5$

$$\begin{vmatrix} 1 & 1 & 3 \\ 2 & -1 & 5 \\ 1 & -3 & 6 \end{vmatrix}$$

$|Z| = 1(-6 + 15) - 1(12 - 5) + 3(-6 + 1)$

$|Z| = 9 - 7 - 15 = 13$

$$Z = \frac{|Z|}{|A|} = \frac{|-13|}{|-5|}$$

$$Z = \frac{13}{5} \quad \text{(D)}$$

27. The minors of a_{22} is ? (a) 1 (b) −1 (c) 2 (d) −2 (e) 3

Solution

Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 3 \\ 1 & -3 & 4 \end{pmatrix}$

Minor of $a_{22} = (1 \times 4) - (1 \times 1)$
$a_{22} = 4 - 1 = 3$ (E)

28. The cofactor of a_{22} is ? (a) 1 (b) -1 (c) 2 (d) -2 (e) 0

Solution

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 3 \\ 1 & -3 & 4 \end{pmatrix}$$

$(1 \times 3)(2 \times 1) = 3 - 2 = 1$
Cofactor $= (-1)^{i+j}(m_{ij})$
$= (-1)^{2+1}(1) = -1$ (B)

29. From the properties of adjoint of a matrix **A**, the inverse of $A = A^{-1}$ is ? (a) $|A| \times AdjA$ (b) $\frac{AdjA}{|A|}$ (c) $\frac{AdjA}{A}$ (d) $A \times AdjA$ (e) None

Solution

$\frac{AdjA}{|A|} = A^{-1}$ (B)

30. If M_{ij} is the minor of a_{ij}, then it,s cofactors is defined as (a) $-M_{ij}$ (b) $a_{ij}M_{ij}$ (c) $-a_{ij}M_{ij}$ (d) $(-1)^{i+j}M_{ij}$ (e) $-(-1)^{i+j}M_{ij}$

Solution

Cofactor $= (-1)^{i+j}(m_{ij})$ (D)

0.5 MATHS 207 2016/2017 EXAMINATION

1. If the equations $x + 3y + z = 0$, $2x - y - z = 0$ *and* $kx + 2y + 3z = 0$ have non trivial solution then **k**= (a) $\frac{13}{2}$ (b) $\frac{9}{2}$ (c) $\frac{-15}{2}$ (d) $\frac{-13}{2}$

Solution

$x + 3y + z = 0$
$2x - y - z = 0$
$kx + 2y + 3z = 0$

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ k & 2 & 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ Find the determinant of A

$|A| = 1(-3 + 2) - 3(6 + k) + 1(4 + k)$
$= -1 - 18 - 3k + 4 + k$
$= -15 - 2k$

$$\begin{pmatrix} 0 & 3 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 3 \end{pmatrix}$$

$|x| = 0$
$x = \frac{|A|}{|x|} = \frac{-15 - 2k}{0} = 0$
$-15 - 2k = 0$
$-15 = 2k$
$k = \frac{-15}{2}$, (C)

2. The equation $x + 2y + 3z = 1$, $2x + y + 3z = 2$ *and* $5x + 5y + 9z = 4$ have (a) No solution (b) Unique solution (c) Infinite solution (d) cannot say anything

3. if $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{pmatrix}$, $A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{3} & \frac{-3}{2} & \frac{1}{2} \end{pmatrix}$, then

(a) $a = 2$, $c = \frac{-1}{2}$ (b) $a = 1$ $c = -1$ (c) $a = -1, c = 1$ (d) $a = \frac{1}{2}$ $c\frac{1}{2}$

Solution

$$I_3 = A.A^{-1} = \begin{bmatrix} 0-4+\frac{10}{3} & 0+3-3 & 0+c+1 \\ \frac{1}{2}-8+5 & \frac{-1}{2}+6-\frac{9}{2} & \frac{1}{2}+2c+\frac{3}{2} \\ \frac{3}{2}-4a+\frac{5}{3} & \frac{-3}{2}+3a-\frac{3}{2} & \frac{3}{2}+ac+\frac{1}{2} \end{bmatrix}$$

$$= \begin{pmatrix} \frac{-2}{3} & 0 & c+1 \\ \frac{-5}{2} & 1 & \frac{4+4c}{2} \\ \frac{19-24a}{6} & \frac{-6+6a}{2} & \frac{4+2ac}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$c = -1$

$\frac{4+2ac}{2} = 1$

$4 + 2ac = 2$

$4 - 2a = 2$

$2a = 2$

$a = 1$

$\therefore\ c = -1\ and\ a = 1$ (B)

4. Consider the following system of equation

$$\begin{aligned} x_1 + x_3 &= 5 \\ x_1 - x_2 - x_3 &= 6 \\ x_2 + x_3 &= 7 \end{aligned}$$

The above system of equation is (a) Inconsistent (b) Consistent with a unique solution (c) Consistent with infinitely many solutions (d) None of the above

Solution

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$$

$|A| = 0 - 0 + 1 = 1$

$$\begin{pmatrix} 5 & 0 & 1 \\ 6 & -1 & -1 \\ 7 & 1 & 1 \end{pmatrix}$$

$|x| = 5(-1+1) - 0(6+7) + 1(6+7) = 13$

$x = \frac{|A|}{|x|} = \frac{1}{13}$ (B)

It is consistent with a unique solution. because the number of equations must be at least equal to the number of variables.

5. $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0,$ then x =

(a) $\frac{3}{2}, \frac{3}{11}$ (b) $\frac{3}{2}, \frac{11}{3}$ (c) $\frac{2}{3}, \frac{11}{3}$ (d) $\frac{2}{3}, \frac{3}{11}$

Solution

Finding the determinant

$(3x-8)[(3x-8)^2 - 9] - 3[(3x-8)(3) - 9] + 3[9 - 3(3x-8)]$

$(3x-8)(9x^2 - 48x + 64 - 9) - 3(9x - 24 - 9) + 3(9 - 9x + 24)$

$(3x-8)(9x^2 - 48x + 55) - 3(9x - 33) + 3(33 - 9x)$

$= 27x^3 - 144x^2 + 165x - 27x^2 + 384x - 440 - 27x + 99 + 99 - 27x$

$= 27x^3 - 216x^2 + 495x - 242$

$\therefore\ x = \frac{2}{3}, \frac{11}{3}$ (C)

6. Let $A = \begin{pmatrix} 4 & 4k & k \\ 0 & k & 4k \\ 0 & 0 & 4 \end{pmatrix}$ If $det(A)^2 = 16$ then $|K|$ is

(a) 1 (b) $\frac{1}{4}$ (c) 4 (d) 4^2

Solution

$$A \times A = \begin{pmatrix} 16+0+0 & 16k+4k^2+0 & 4k+16k^2+4k \\ 0+0+0 & 0+k^2+0 & 0+4k^2+16k \\ 0+0+0 & 0+0+0 & 0+0+16 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 16 & 4k^2+16k & 16k^2+8 \\ 0 & k^2 & 4k^2+16k \\ 0 & 0 & 16 \end{pmatrix}$$

$= 16(16k^2) - 4k^2 - 16k(0)$

$= k^2 = \frac{1}{16} = \frac{1}{4}$ (B)

7. If the equation $x - 2y + 3z = 0,\quad -2x + 3y + 2z = 0\ and\ -8x + \lambda y = 0$ have non-trivial solution then $\lambda =$ (a) 18 (b) 13 (c) -10 (d) 4

Solution

$$\begin{pmatrix} 1 & -2 & 3 \\ -2 & 3 & 2 \\ -8 & \lambda & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$